Study Guide Answers

1. $x = 2$, $x = -\dfrac{1}{3}$
2. $x \approx 0.45$
3. the sides of the rectangle have lengths of 6 and 8
4. the sides of the rectangle have lengths of 2 and 4
5. the trains meet at about 3:18 PM and about 318.26 miles from Chicago
6. 23.05 miles
7. $2x + 1 + h$
8. $-2 < x < 2$
9. (a) $x \geq -2$
   (b) $f \circ g = x^2 + \sqrt{x^2 + 2}$
10. $f^{-1}(x) = -\dfrac{4x + 5}{7x - 2}$
11. $f^{-1}(x) = \dfrac{1}{2}\left(1 + \dfrac{\ln x}{\ln 3}\right)$
12. -2
13. When $f(x)$ is divided by $g(x)$, the remainder is 0:

$$\begin{array}{r} x^3 \qquad\quad -x+1 \\ x^2+1\overline{)\;x^5 \qquad +x^2-x+1} \\ \underline{-x^5 \quad -x^3} \\ -x^3+x^2-x \\ \underline{x^3 \qquad\quad +x} \\ x^2 \qquad\quad +1 \\ \underline{-x^2 \qquad\quad -1} \\ 0 \end{array}$$

14. $g(x) = -3(x + 2) - 3\sqrt{x + 1} - 3$
15. One series of transformations is: (1) expand vertically by a factor of 3, (2) reflect across the x-axis, (3) shift 2 units upward, (4) shift 2 units to the left.
16. One series of transformations is: (1) expand vertically by a factor of 2, (2) shift 1 unit to the right.
17. 0, $-\dfrac{2}{3}$, $\dfrac{1}{2}$, -1
18. $\dfrac{1}{2}$, $2 + i$, $2 - i$
19. (a) $x = 0$
    (b) $y = 0$
    (c) $x = -2$
    (d) $y = 1$
20. (a) none
    (b) $x = -2$
    (c) $x = 1$
    (d) $y = 1$
21. $x \geq \dfrac{3}{2}$
22. $x \leq -4$ or $x > -\dfrac{3}{2}$
23. (a) $-i$
    (b) $-i$
    (c) $x^{7/3}y^{-17/3}$
    (d) $\ln\left(\dfrac{e(x+1)}{2x}\right)$
    (e) -1
    (f) $x + 1 - x^2$
24. (a) \$1082.86
    (b) \$1083.14
    (c) \$1083.28
    (d) \$1083.29
25. (a) 4.89%
    (b) 4.88%
26. (a) 4.28 feet
    (b) 24.81 years
27. 3195.51 years
28. 3
29. $x = \dfrac{3\ln 5}{\ln 5 - \ln 3}$
30. $\dfrac{3}{5}$
31. none
32. $\dfrac{2\pi}{5}$
33. $\dfrac{\pi}{6} + 2\pi$, $\dfrac{\pi}{6} - 2\pi$, $\dfrac{\pi}{6} + 4\pi$
34. (a) $\dfrac{\sqrt{2}}{2}$
    (b) $\dfrac{1}{2}$

(c) 0

(d) 2

(e) 0

(f) $\sqrt{3}$

(g) 1

(h) $-\dfrac{\sqrt{2}}{2}$

(i) $-\sqrt{3}$

(j) $-\sqrt{2}$

35. (a) $-\dfrac{\sqrt{7}}{4}$

(b) $\dfrac{\sqrt{7}}{3}$

(c) $-\dfrac{4}{3}$

(d) $\dfrac{3}{4}$

36. (a) $-\dfrac{2\sqrt{2}}{3}$

(b) $-2\sqrt{2}$

(c) 3

(d) $\dfrac{1}{3}$

37. amplitude $= 4$, period $= \dfrac{2\pi}{5}$, phase shift $= \dfrac{\pi}{10}$

38. $A = 2$, $b = \dfrac{\pi}{2}$, $c = \dfrac{\pi^2}{6}$

39. (a) $f(t) = -2\sin 4t$

(b) $f(t) = 3\sin\left(4t - \dfrac{\pi}{3}\right)$

40. (a) false. $\sin 2x = 2\sin x \cos x$

(b) true. $\cos x \tan x = \cos x \cdot \dfrac{\sin x}{\cos x} = \sin x$

(c) false.

$$\sin(x+\pi) = \sin x \cos \pi + \cos x \sin \pi = -\sin x$$

(d) false. counterexample:

$$\tan \frac{\pi}{4} + \cot \frac{\pi}{4} = 1 + 1 = 2$$

(e) false. counterexample:

$$\cos^2 \frac{\pi}{3} = \frac{1}{4} \quad \text{but} \quad \cos^2 \frac{\pi}{6} - \sin^2 \frac{\pi}{6} = \frac{1}{2}$$

(f) false. $\tan x = \dfrac{\sin x}{\cos x}$

41.

$$\begin{aligned}
\frac{\cot\theta - \tan\theta}{\sin\theta + \cos\theta} &= \frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\sin\theta + \cos\theta} \\
&= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta(\sin\theta + \cos\theta)} \\
&= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\sin\theta\cos\theta(\cos\theta + \sin\theta)} \\
&= \frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta} \\
&= \frac{1}{\sin\theta} - \frac{1}{\cos\theta} \\
&= \csc\theta - \sec\theta
\end{aligned}$$

42.

$$\begin{aligned}
\frac{1 + \csc\beta}{\cot\beta + \cos\beta} &= \frac{1 + \frac{1}{\sin\beta}}{\frac{\cos\beta}{\sin\beta} + \cos\beta} \\
&= \frac{\sin\beta + 1}{\cos\beta + \sin\beta\cos\beta} \\
&= \frac{1 + \sin\beta}{\cos\beta(1 + \sin\beta)} \\
&= \frac{1}{\cos\beta} \\
&= \sec\beta
\end{aligned}$$

43.

$$\begin{aligned}
\frac{\tan^2 x}{\sec x + 1} &= \frac{\sec^2 x - 1}{\sec x + 1} \\
&= \frac{(\sec x + 1)(\sec x - 1)}{\sec x + 1} \\
&= \sec x - 1 \\
&= \frac{1}{\cos x} - 1 \\
&= \frac{1 - \cos x}{\cos x}
\end{aligned}$$

44. (a) $\dfrac{\sqrt{2} + \sqrt{6}}{4}$

(b) $-\dfrac{\sqrt{2} + \sqrt{6}}{4}$

(c) $\dfrac{\sqrt{2 + \sqrt{3}}}{2}$

(d) $\dfrac{\sqrt{2 - \sqrt{2}}}{2}$

45. (a) $\dfrac{2\sqrt{10}}{7}$

(b) $\dfrac{2}{\sqrt{14}}$

(c) $-\dfrac{31}{49}$

46. (a) $\dfrac{\sqrt{15}}{4}$
    (b) does not exist
    (c) $\dfrac{\pi}{6}$
    (d) $-\dfrac{\sqrt{77}}{2}$
    (e) $\dfrac{5}{2\sqrt{6}}$

47. $x = \pm\dfrac{\pi}{32} + \dfrac{k\pi}{4}$ for $k = 0, \pm1, \pm2, \ldots$

48. $x = \dfrac{7\pi}{6} + 2k\pi,\ -\dfrac{\pi}{6} + 2k\pi$ for $k = 0, \pm1, \pm2, \ldots$

49. (a) $a = \sqrt{34}$, $B = 30.96°$, $C = 59.04°$
    (b) $a = 3.41$, $b = 2.23$, $A = 80°$
    (c) $a = 7.39$, $B = 39.45°$, $C = 30.55°$

50. 2.64 miles

51. distance to 1st and 3rd base is 63.72 feet; distance to 2nd base is 66.78 feet

52. (a) 2
    (b) $z = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$
    (c) $z^{12} = 2^{12}\left(\cos 4\pi + i\sin 4\pi\right)$
    (d) $(2)^{1/4}\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)$,
    $(2)^{1/4}\left(\cos\dfrac{7\pi}{12} + i\sin\dfrac{7\pi}{12}\right)$,
    $(2)^{1/4}\left(\cos\dfrac{13\pi}{12} + i\sin\dfrac{13\pi}{12}\right)$,
    $(2)^{1/4}\left(\cos\dfrac{19\pi}{12} + i\sin\dfrac{19\pi}{12}\right)$

53. $\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}$,
    $\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}$,
    $\cos\dfrac{9\pi}{6} + i\sin\dfrac{9\pi}{6}$

54. (a) $||\overrightarrow{\mathbf{u}}|| = \sqrt{13}$, $||\overrightarrow{\mathbf{v}}|| = \sqrt{5}$
    (b) $2\overrightarrow{\mathbf{u}} - 3\overrightarrow{\mathbf{v}} = \langle 7, 12\rangle$
    (c) $\left\langle \dfrac{2}{\sqrt{13}}, \dfrac{3}{\sqrt{13}} \right\rangle$

55. (a) $\overrightarrow{PQ} = \langle -5, -4\rangle$
    (b) $||\overrightarrow{PQ}|| = \sqrt{41}$
    (c) $\left\langle \dfrac{5}{\sqrt{41}}, \dfrac{4}{\sqrt{41}} \right\rangle$