

MATH 180 Final 05/02/02

1	5	9
2	6	10
3	7	11
4	8	12

Total _____

Name (print) _____

Section (circle) Colman Doyle 9AM Doyle 1PM Liu Lowndes Radford Ruan
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Tu/Th Discussion (circle) 8 9 10 11 12 1 2 3

(1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *You must show* your work for credit. (4) There are *twelve questions* on this exam. **Check to see that this copy is complete.** (5) If you use a calculator it must be *your own*. (6) Round decimal answers to *four decimal places*. (7) *You are expected to abide by the University's rules concerning academic honesty.*

1. (15 points) Let $f(x) = \begin{cases} kx + 8 & : x \leq -2 \\ kx^2 - 4 & : x > -2 \end{cases}$.

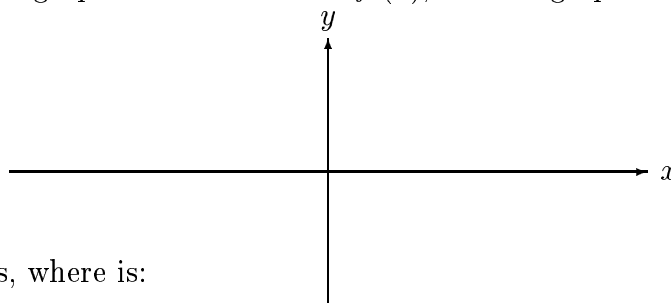
- a) Determine the value of k which makes the function $y = f(x)$ continuous for all real numbers.
- b) Sketch the graph of $y = f(x)$ using this value of k .

2. (15 points) Using the definition of derivative as the limit of the difference quotient, compute the derivative of $f(x) = 3x^2 - x$.

3. (15 points) Consider the equation $x^2 + 3xy + 3y^2 = 7$.

- a) Find dy/dx in terms of x and y .
- b) Find an equation of the line tangent to the curve determined by the equation at $(1, 1)$.

4. (15 points) Below is the graph of the derivative $f'(x)$; *not* the graph of $f(x)$.



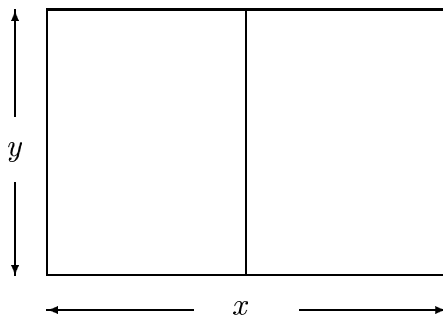
Among the labeled points, where is:

- a) $f'(x)$ greatest? b) $f'(x)$ negative? c) $f'(x)$ least? d) $f(x)$ a local minimum? e) $f''(x)$ positive?

5. (30 points) Find the derivative of the following three functions. *The differentiation rules you use must be implicit in your calculations; do NOT simplify answers.*

a) $(x^3 - \frac{7}{x} + 6)^{298} \ln(\cos x)$ b) $\frac{7^x}{\sqrt{x-6}}$ c) $e^{-9x^3 + \pi x} \sinh(2x)$.

6. (20 points) Let $f(x) = x^4 - 3x^2$. You must use Calculus to derive your answers below.
- Determine the x -coordinates of the critical points of $y = f(x)$.
 - Determine the intervals on which $f(x)$ is increasing, on which $f(x)$ is decreasing.
 - Determine where the graph of $f(x)$ is concave up, where the graph of $f(x)$ is concave down.
 - Determine the inflection points (x, y) on the graph of $y = f(x)$.
7. (10 points) Find $\lim_{x \rightarrow 0} (e^x - 1)/(\sin 3x)$, if the limit exists, by L'Hopital's rule.
8. (12 points) The volume of a spherical balloon is $V = (4/3)\pi r^3$, where r its radius. The radius is increasing at the rate of 0.5 inches per second. Find the rate at which the volume is changing when $r = 11$ inches, and indicate the units of the rate of change of volume.
9. (13 points) Let $f(x) = (x^2 + 9)^{3/2}$.
- Find the line tangent line approximation (local linearization) to $f(x)$ near $x = 4$.
 - Use this tangent line approximation to estimate $f(4.1)$.
10. (20 points) Let $y = f(x) = 4x^3 + 2$.
- Use the Fundamental Theorem of Calculus to find the area of the region in the plane bounded by the graph of $y = f(x)$, the x -axis, and the vertical lines $x = 1$, $x = 3$. *How the Fundamental Theorem is used must be clear from your calculations.*
 - Find the average value of $f(x)$ on the interval $[1, 3]$.
11. (15 points) Find the left and right sums based on 3 equal subdivisions which estimate $\int_0^6 2^x dx$. You must write out the terms of the sums as well as calculate what the sums add up to.
12. (20 points) A 1,000 square foot rectangular garden plot x feet long and y feet wide is to be divided into two parts and each part is to be fenced. Fencing for the two horizontal lengths x feet long costs \$5 per foot and fencing for the three vertical lengths y feet long costs and \$10 per foot.



- Find the total cost of the fencing C in terms of x and find the domain of $C = C(x)$.
- Find the dimensions (x and y) of the garden which *minimize* the total cost of the fencing. *You must justify your answer.*