

1. (15 points)
$$f(x) = \begin{cases} kx + 8 & : x \leq -2 \\ kx^2 - 4 & : x > -2 \end{cases} .$$

a) The value of k we seek is the one which makes the one-sided limits $\lim_{x \rightarrow -2^-} f(x) = k(-2) + 8$ and $\lim_{x \rightarrow -2^+} f(x) = k(-2)^2 - 4 = 4k - 4$ equal. Thus $-2k + 8 = 4k - 4$ (**4pts**), or $k = 2$ (**4pts**).

b) Graph of $y = -2x + 8$, where $x \leq -2$ (**3pts**), together with graph of $y = 4x^2 - 8$, where $x \geq -2$ (**4pts**).

2. (15 points) $f(x) = 3x^2 - x$. Thus

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \text{ (5pts)} \\ &= \lim_{h \rightarrow 0} \frac{[3(a+h)^2 - (a+h)] - [3a^2 - a]}{h} \\ &= \lim_{h \rightarrow 0} \frac{[3(a^2 + 2ah + h^2) - (a+h)] - [3a^2 - a]}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(2ah + h^2) - h}{h} \\ &= \lim_{h \rightarrow 0} 6a + 3h - 1 \text{ (calculations, 9pts)} \\ &= 6a - 1 \text{ (1pt)}. \end{aligned}$$

3. (15 points) $x^2 + 3xy + 3y^2 = 7$. Differentiating both sides with respect to x yields

a) $2x + 3(y + x \frac{dy}{dx}) + 3(2y \frac{dy}{dx}) = 0$ (**4pts**), and thus $\frac{dy}{dx} = -\frac{2x + 3y}{3x + 6y}$ (**4pts**).

b) $\frac{dy}{dx} = -\frac{5}{9}$ (**4pts**) when $(x, y) = (1, 1)$. Thus $y - 1 = -\frac{5}{9}(x - 1)$, or $y = -\frac{5}{9}x + \frac{14}{9}$ (**3pts**).

4. (15 points) a) B, G; or B; or G (**3pts**) b) D, E (**3pts**) c) D (**3pts**) d) A (**3pts**) e) A, E, F (**3pts**)

5. (30 points) Answers unsimplified. **Point value for parts in boldface.**

a) $((x^3 - \frac{7}{x} + 6)^{298} \ln(\cos x))' =$
 $[298(x^3 - \frac{7}{x} + 6)^{297}(\mathbf{2})(3x^2 + \frac{7}{x^2})(\mathbf{2})] \ln(\cos x)(\mathbf{1}) + (x^3 - \frac{7}{x} + 6)^{298}(\mathbf{2})[(\frac{1}{\cos x})(\mathbf{2})(-\sin x)(\mathbf{1})].$

b) $(\frac{7^x}{\sqrt{x} - 6})' = \frac{[7^x \ln 7](\mathbf{2})(\sqrt{x} - 6)(\mathbf{2}) - 7^x(\mathbf{2})[\frac{1}{2\sqrt{x}}](\mathbf{2})}{(\sqrt{x} - 6)^2(\mathbf{2})}.$

c) $(e^{-9x^3 + \pi x} \sinh(2x))' = [e^{-9x^3 + \pi x}(\mathbf{2})(-27x^2 + \pi)(\mathbf{2})] \sinh(2x)(\mathbf{1}) + e^{-9x^3 + \pi x}(\mathbf{2})[(\cosh(2x))(\mathbf{2})2(\mathbf{1})].$

6. (20 points) $f(x) = x^4 - 3x^2$; $f'(x) = 4x^3 - 6x = 2x(2x^2 - 3)$, $f''(x) = 12x^2 - 6 = 6(2x^2 - 1)$.
- a) x -coordinates of the critical points: $x = 0$ (2pts), $-\sqrt{3/2}$ (2pts), $\sqrt{3/2}$ (2pts).
- b)

dec	inc	dec	inc	$f(x)$	derived from test values. $f(x)$ is <i>increasing</i> on $[-\sqrt{3/2}, 0]$ (1pt), $[\sqrt{3/2}, \infty)$ (1pt); $f(x)$ is <i>decreasing</i> on $(-\infty, -\sqrt{3/2}]$; (1pt), $[0, \sqrt{3/2}]$ (1pt).		
(-)	$-\sqrt{3/2}$	(+)	0	(-)		$\sqrt{3/2}$	(+)
- c)

concave up	concave down	concave up	$f(x)$	derived from test values. Graph of $f(x)$ is concave <i>up</i> on $(-\infty, -\sqrt{1/2})$ (2pts), $(\sqrt{1/2}, \infty)$ (2pts); <i>down</i> on $(-\sqrt{1/2}, \sqrt{1/2})$ (2pts).	
(+)	$-\sqrt{1/2}$	(-)	$\sqrt{1/2}$		(+)
- d) Inflection points: $(-\sqrt{1/2}, -5/4)$ (2pts), $(\sqrt{1/2}, -5/4)$ (2pts).
7. (10 points) L'Hopital's rule does apply and $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 3x} = \lim_{x \rightarrow 0} \frac{e^x}{3 \cos 3x}$ (5pts) = $\frac{1}{3}$ (5pts).
8. (12 points) Since $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dt} = \frac{4}{3}\pi(3r^2 \frac{dr}{dt}) = 4\pi r^2 \frac{dr}{dt}$ (6pts). Given that $r = 11$ inches and $\frac{dr}{dt} = .5$ (1pt) inches/second, we have $\frac{dV}{dt} = 4\pi 11^2 (.5)$ (3pts) = 242π cubic inches/second (2pts).
9. (13 points) $f(x) = (x^2 + 9)^{3/2}$.
- a) $f'(x) = \frac{3}{2}(x^2 + 9)^{1/2}(2x) = (x^2 + 9)^{1/2}3x$ (4pts); thus $f'(4) = 60$. $y = f'(4)(x - 4) + f(4) = 60(x - 4) + 125$; $f(x) \approx 60(x - 4) + 125$ (4pts).
- b) $f(4.1) \approx 60(4.1 - 4) + 125 = 6 + 125 = 131$ (5pts).
10. (20 points) $y = f(x) = 4x^3 + 2$.
- a) Area = $\int_1^3 (4x^3 + 2) dx$ (5pts) = $\int_1^3 (x^4 + 2x)' dx$ (5pts) = $[3^4 + 2(3)] - [1^4 + 2(1)] = 84$ (5pts).
- b) Average value = $84/(3 - 1) = 42$ (5pts).
11. (15 points) $\Delta x = (6 - 0)/3 = 2$. $LHS = (2^0 + 2^2 + 2^4)2$ (4pts) = 42 (3pts), $RHS = (2^2 + 2^4 + 2^6)2$ (4pts) = 168 (4pts). **Numerical answers only, 3 points total.**
12. (20 points) Garden: 1,000 square foot rectangular garden plot x feet long and y feet wide. Fencing for two lengths x feet, \$5 per foot, for three lengths y feet, \$10 per foot.
- a) Cost function $C = C(x) = 2(5x) + 3(10y) = 10x + 30(\frac{1000}{x})$ (6pts), domain $0 < x$ (2pts).
- b) $C'(x) = 10 - \frac{30000}{x^2} = 10\frac{x^2 - 3000}{x^2}$. Thus $C'(x) = 0$ if and only if $x = \sqrt{3000}$ (4pts).
 Since

dec	inc	$f(x)$	it follows that $C(\sqrt{3000})$ is a minimum for the function $C(x)$ (4pts). Thus $x = \sqrt{3000}$, $y = \frac{1000}{\sqrt{3000}}$ are the dimensions (4pts).
0	$-\sqrt{3000}$	+	