MATH 215 **Summer 2009** **Radford**

Written Homework #6 (REVISED)

Due at the beginning of class 07/24/2009

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1. Let A, B, and C be sets.

(a) Prove $A \cap B \subseteq A \cap C$ and $A \cup B \subseteq A \cup C$ implies $\boxed{B \subseteq C}$.

(b) Use part (a) to show that $A \cap B = A \cap C$ and $A \cup B = A \cup C$ implies $\boxed{B = C}$.

2. Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow X$ be functions which satisfy $g \circ f = \mathrm{I}_X$. Prove that f is injective and g is surjective.

3. Let $f : [1/2, \infty) \longrightarrow [-1/4, \infty)$ be defined by $f(x) = x^2 - x$ for all $x \in [1/2, \infty)$.

(a) Show that f is injective.

(b) Show that f is surjective.

(c) Find f^{-1}.

4. Let $f : X \longrightarrow Y$ be a function and $G_f \subseteq X \times Y$ be its graph. Show that f is bijective if and only if $G_f^{op} = \{(y, x) \mid (x, y) \in G_f\}$ is the graph of a function. [*Hint*: You may use the fact that $G \subseteq X \times Y$ is the graph of a function if and only if (a) $\forall x \in X, \exists y \in Y, (x, y) \in G$ and (b) $(x, y), (x, y') \in G$ implies $y = y'$.

5. Let U be a universal set and $A, B \subseteq U$. Recall that $\chi_\emptyset = 0$, $\chi_U = 1$, and $\chi_A = \chi_B$ if and only if $A = B$.

(a) Complete the table

$x \in A$	$x \in B$	$\chi_A(x)$	$\chi_B(x)$	$\chi_{A \cap B}(x)$	$\chi_A(x)\chi_B(x)$
T	T	1	1		
T	F	1	0		
F	T				
F	F				

and explain how it proves that $\chi_{A \cap B} = \chi_A \chi_B$.

(b) Complete the table

$x \in A$	$x \in B$	$\chi_A(x)$	$\chi_B(x)$	$\chi_{A \cup B}(x)$	$\chi_A(x) + \chi_B(x) - \chi_{A \cap B}(x)$
T	T	1	1		
T	F	1	0		
F	T				
F	F				

and explain how it proves that $\chi_{A \cup B} = \chi_A + \chi_B - \chi_{A \cap B}$.