Math 180 **Final Exam** **December 8, 2005**

Show all your work. An unjustified answer is not correct.

1. Differentiate with respect to x. Write your answers showing the use of the appropriate techniques. Do <u>*not*</u> simplify.

$$\text{(a) } x^{1066} + x^{1/2} - x^{-2}, \qquad \text{(b) } e^{\sqrt{x}}, \qquad \text{(c) } \frac{\sin(x)}{5+x^2}.$$

2. Differentiate, writing your answers as in problem 1.

$$\text{(a) } e^{3x}\cos(5x), \qquad \text{(b) } \ln(x^2+x+1), \qquad \text{(c) } \tan\left(\frac{1}{x}\right).$$

3. Use calculus to find the exact x- and y-coordinates of any local maxima, local minima, and inflection points of the function $f(x) = x^3 - 12x + 5$.

4. Use implicit differentiation to find the slope of the line tangent to the curve

$$x^2 + xy + y^2 = 7$$

at the point $(2, 1)$.

5. Estimate the integral $\int_0^{40} f(t)\,dt$ using the left Riemann sum with four subdivisions. Some values of the function f are given in the table:

t	0	10	20	30	40
$f(t)$	5.3	5.1	4.6	3.7	2.3

If the function f is known to be decreasing, could the integral be larger than your estimate? Explain why or why not?

6. Write the integral which gives the area of the region between $x = 0$ and $x = 2$, above the x-axis, and below the curve $y = 9 - x^2$.
Evaluate your integral exactly to find the area.

7. Find the average value of the function $f(x) = \dfrac{1}{x^2}$ on the interval $2 \le x \le 6$.

8. Find

$$\lim_{x\to 0} \frac{1-\cos(3x)}{x^2}.$$

Explain how you obtain your answer.

9. The function $f(x)$ has the following properties:
   - $f(5) = 2$,
   - $f'(5) = 0.6$,
   - $f''(5) = -0.4$.

   (a) Find the tangent line to $y = f(x)$ at the point $(5, 2)$.

   (b) Use (a) to estimate $f(5.2)$.

   (c) If f is known to be concave down, could your estimate in (b) be greater than actual $f(5.2)$? Give a reason supporting your answer.

10. The point (x, y) lies on the curve $y = \sqrt{x}$.

    (a) Find the distance from (x, y) to $(2, 0)$ as a function $f(x)$ of x alone.

    (a) Find the value of x that makes this distance the smallest.