**MATH 180** # Final Examination **Thursday, May 4, 2006**

**Name (print)** ____________________ **Signature** ______________________

**UIC ID** ______________________________

(1) *Write* your name and your instructor's name on your examination booklet. (2) *Write* your answers in the exam booklet provided. (3) *Return* your copy of the examination with the examination booklet(s). (4) *Show* your work. **An unjustified answer receives no credit**. (4) There are **10** questions on this examination. Check to see that this copy is complete. (5) If you use a calculator it must be *your own.* (6) *You are expected to abide by the University's rules concerning academic honesty.*

(7) *Circle* your instructor's name and lecture hour:

Dias (8AM) Kobotis (9AM) Chakravorty (10AM) Radford (11AM) Masley (12PM)

Kashcheyeva (1PM) Tartakoff (1PM) Chakravorty (2PM) Ruan (3PM)

(8) *Circle* your discussion hour: 8AM 9AM 10AM 11AM 12PM 1PM 2PM 3PM

Do not write in this area.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| **30** | **15** | **20** | **20** | **20** | **25** | **15** | **15** | **20** | **20** |
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**SCORE** ____________ **/200**

1. (30) Find the derivatives of the following five functions; *do NOT simplify answers.*

$$(a)\ 3^x + x^3 + \frac{x}{3} + \frac{3}{x} \qquad (b)\ \frac{x^4 - \cos x}{2x^6 + \tan x} \qquad (c)\ \sqrt{(\ln x)^2 + 5}$$

$$(d)\ (x^2+1)^{231}(1+\sin x) \qquad (e)\ e^{3x} + \sinh 4x + \arctan x + \frac{1}{\sqrt{x}}$$

2. (15) Let $f(x) = 2x^2 - x - 1$. Use the definition of the derivative as the limit of a difference quotient to compute $f'(x)$.

3. (20) The height of a ball thrown straight up into the atmosphere on a strange and exotic planet at time $t$ seconds is $s(t) = -6t^2 + 3t + 30$ meters. *Use Calculus to answer the following questions.*

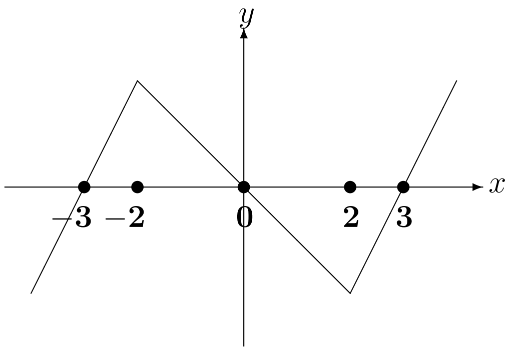
(a) Find the velocity and the acceleration of the ball at time $t$. Give the units for each.

(b) When does the ball reach its maximum height?

(c) What is its maximum height?

(d) When does the ball hit the ground?

4. (20) The graph below is that of the *derivative* $f'(x)$. The domain of $f(x)$ is $(-4, 4)$.




(a) Find the values of $x$ where a local minimum of $f(x)$ occurs.

(b) Find the intervals on which the graph of $y = f(x)$ is concave down.

(c) Find the $x$-coordinates of inflection points on the graph of $y = f(x)$.

5. (20) The function $y = f(x)$ is defined implicitly by the equation $y^3x^2 + xy - x^2 = 2$.

(a) Find an equation of the line tangent to the graph of $y = f(x)$, that is line tangent to the curve defined by the equation, at the point $(2, 1)$.

(b) Use tangent line approximation to estimate $f(2.1)$.

6. (25) Let $f(x) = x^4 + x^3$. Part (e) of this problem is on the next page.

(a) Find the *critical points* of $y = f(x)$.

(b) Find the intervals on which $f(x)$ is *increasing*, on which $f(x)$ is *decreasing*.

(c) Find the intervals on which the graph of $y = f(x)$ is *concave down*, on which the graph is *concave up*.

(d) Find the *inflection points* on the graph of $y = f(x)$.

(e) Using the information derived from parts (a)–(d), sketch the graph of $y = f(x)$. Plot the points corresponding to local maxima and minima, inflection points, and points where the graph crosses the $x$-axis. *Coordinates of points must be expressed as integers or fractions, like* $-31$ or $7/11$.

7. (15) Find:

(a) $\displaystyle\lim_{x\longrightarrow 0} \frac{e^{3x} - 1 - 3x}{x^2}$ by L'Hopital's rule;

(b) the unique real number $c$ such that the function given by $f(x) = \begin{cases} x^2 + 4x & : \; x \leq 3 \\ 2cx + 1 & : \; 3 < x \end{cases}$ is continuous at $x = 3$.

8. (15) Let $f(x) = x^2 + x$.

(a) Find the right hand sum (right Riemann sum) for $\displaystyle\int_0^2 (x^2 + x)\, dx$ where $n = 4$.

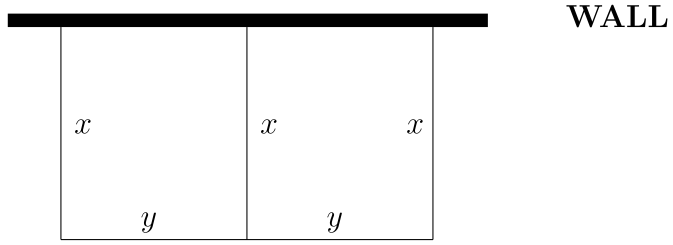
(b) Is the right hand sum of part (a) an underestimate or is it an overestimate of $\displaystyle\int_0^2 (x^2 + x)\, dx$? Explain your answer.

9. (20) *Use Calculus* to find:

(a) the average value of the function $g(x) = x^3$ on the interval $[-1, 3]$;

(b) the area of the region bounded by the lines $x = 0$, $x = \ln 2$, the $x$-axis, and the graph of $y = f(x) = e^x + 2$.

10. (20) A farmer wishes to construct two rectangular fenced pens of the same dimensions against a long wall as depicted in the diagram:




The area of *each* pen is to be 5,000 square feet. Fencing for the three sides perpendicular to the wall costs \$25 per foot and fencing for the side parallel to the wall costs \$12 per foot.

(a) Express the total cost of the fencing $C(x)$ as a function of $x$ and find its domain.

(b) Find the $x$ such that the cost $C(x)$ is minimal. *Your answer must be justified by Calculus.*

(c) Find the minimal cost of the fencing.