MATH 180 # Hour Exam I (and Solution) **Radford**

02/10/06

Name (print) ____________________ Tu/Th Discussion (circle) 11 12 1

(1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *Show* your work. (4) There are *six questions* on this exam. (5) If you use a calculator it must be *your own*. (6) Round decimal answers to *four decimal places*. (7) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (20 points) The population $P(t)$, where t is years, of a small town is growing exponentially. Given that $P(2) = 1600$ and $P(3) = 2800$, find:

(a) $P(t)$,

Solution: $P(t) = P_0 a^t$ for some constants P_0, a. Since $\dfrac{P(3)}{P(2)} = \dfrac{P_0 a^3}{P_0 a^2} = a$ we have $a = \dfrac{2800}{1600} = \boxed{\dfrac{7}{4} = 1.75 \quad (\textbf{4 points}).}$ Since $1600 = P(2) = P_0 a^2 = P_0(1.75)^2$ it follows that $P(t) = \boxed{\dfrac{1600}{1.75^2}(1.75)^t \approx (522.4490)1.75^t \quad (\textbf{4 points}).}$

(b) $P(5)$,

Solution: $P(5) = \boxed{8575\ (\textbf{4 points}).}$

(c) the *annual* percentage growth rate, and

Solution: $a = 1.75 = 1 + .75$ so the annual percentage growth rate is $100(.75)\% = \boxed{75\%\ (\textbf{4 points}).}$

(d) the *continuous* percentage growth rate.

Solution: Set $e^k = a = 1.75$. The continuous percentage growth rate is $100k\% = \boxed{100(\ln 1.75)\% \approx 55.9616\%\ (\textbf{4 points}).}$

2. (15 points) Let $y = f(x) = 7x^2 - 3x + 2$. Starting with the difference quotient, use algebra to calculate $f'(x)$.

Solution:

$$\begin{aligned} f'(x) &= \boxed{\lim_{h \longrightarrow 0} \frac{[7(x+h)^2 - 3(x+h) + 2] - [7x^2 - 3x + 2]}{h} \quad (\textbf{6 points})} \\ &= \lim_{h \longrightarrow 0} \frac{[7(x^2 + 2xh + h^2) - 3(x+h) + 2] - [7x^2 - 3x + 2]}{h} \end{aligned}$$

$$= \lim_{h\longrightarrow 0} \frac{7(2xh+h^2)-3h}{h}$$

$$= \lim_{h\longrightarrow 0} \frac{(14x+7h-3)h}{h}$$

$$= \lim_{h\longrightarrow 0} 14x+7h-3 \text{ (\textbf{6 points for calculations to this point})}$$

$$= \boxed{14x-3 \text{ (\textbf{3 points}).}}$$

3. (18 points) Sketch the graph of a function $y = f(x)$ which has *all* of the following properties:

(a) $\lim_{x\to 4} f(x) = \infty = \lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x) = -\infty$, $\lim_{x\to -\infty} f(x) = 0 = \lim_{x\to\infty} f(x)$,

(b) $f'(x) > 0$ on the intervals $(-2,0)$, $(0,4)$,

(c) $f'(x) < 0$ on the interval $(-\infty,-2)$, $(4,\infty)$,

(d) $f''(x) > 0$ on the intervals $(-3,0)$ and $(3,4)$, $(4,\infty)$,

(e) $f''(x) < 0$ on the interval $(-\infty,-3)$, $(0,3)$.

You must label the numbers $-3,-2,0,3,4$ on your x-axis.

Solution: Asymtotes, increase/decrease, concave up/down (**6 points each**). See the file "Hour Exam I Graph".

Note: Since the derivative exists at all numbers except for $0,4$, and differentiability implies continuity, there are no breaks in the graph of $y = f(x)$ except at $0,4$ possibly (where there are indeed breaks).

4. (17 points) Let $f(x) = 3x + \dfrac{5}{x+2}$. Then $f'(x) = 3 - \dfrac{5}{(x+2)^2}$.

(a) Find an equation to the line tangent to the graph of $y = f(x)$ at $x = 2$.

Solution: $\boxed{f'(2) = \frac{43}{16} = 2.6875 \text{ (\textbf{6 points}).}}$ Since $f(2) = \dfrac{29}{4}$, *an* equation of the tangent line is $\boxed{y - \frac{29}{4} = \frac{43}{16}(x-2) \text{ (\textbf{6 points}).}}$

(b) Assume that $f(x)$ is the total cost of producing x items. Using the derivative, *approximate* the cost of producing the 9^{th} item.

Solution: $\boxed{f'(8) = 2.95 \text{ (\textbf{5 points}).}}$

OVER FOR THE TWO REMAINING PROBLEMS

5. (15 points) Let $f(x) = \begin{cases} 3x^2 + 2a & : \quad x \leq 2 \\ 4x - 15 & : \quad x > 2 \end{cases}$. Compute:

(a) $\lim_{x \to 2^-} f(x)$,

Solution: $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 3x^2 + 2a = \lim_{x \to 2} 3x^2 + 2a = \boxed{12 + 2a \text{ } (\textbf{5 points})}$ since $g(x) = 3x^2 + 2a$ is a continuous function.

(b) $\lim_{x \to 2^+} f(x)$, and

Solution: $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 4x - 15 = \lim_{x \to 2} 4x - 15 = \boxed{-7 \text{ } (\textbf{5 points})}$ since $g(x) = 4x - 15$ is a continuous function.

(c) a such that $y = f(x)$ is continuous at $x = 2$.

Solution: We need $f(2) = \lim_{x \to 2} f(x)$; that is $f(2) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$ or equivalently $12 + 2a = 12 + 2a = -7$. Thus $\boxed{a = -\frac{19}{2} \text{ } (\textbf{5 points}).}$

6. (15 points) A particle moving along a straight line has position $s(t) = t^3 - 12t^2 - 60t$ at time t, where $t \geq 0$. Given that $s'(t) = 3t^2 - 24t - 60$ and $s''(t) = 6t - 24$, find:

(a) when the particle is moving to the right,

Solution: $s'(t) = v(t) > 0$ and $t \geq 0$, or equivalently $3(t^2 - 8t - 20) = 3(t - 10)(t + 2) > 0$ and $t \geq 0$. $\boxed{10 < t \text{ } (\textbf{5 points}).}$ Acceptable also is $10 \leq t$.

(b) when it is moving to the left, and

Solution: $s'(t) = v(t) < 0$ and $t \geq 0$. From the calculations for part a) we have $\boxed{0 \leq t < 10 \text{ } (\textbf{5 points}).}$ Acceptable also is $0 \leq t \leq 10$.

(c) when its velocity is decreasing.

Solution: $s''(t) = v'(t) < 0$ and $t \geq 0$, or equivalently $6t - 24 = 6(t - 4) < 0$ and $t \geq 0$. $\boxed{0 \leq t < 4 \text{ } (\textbf{5 points}).}$ Acceptable also is $0 \leq t \leq 4$.