MATH 180 # Hour Exam II **Radford 04/07/06**

(Solution Included)

Name (print) ______________________ **Tu/Th Discussion (circle)** 11 12 1

(1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *Show* your work. (4) There are *six questions* on this exam. (5) If you use a calculator it must be *your own.* (6) Round decimal answers to *four decimal places.* (7) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (20 points) Find the derivatives of the following functions; *do NOT simplify answers.*

a) $x^8 + 8^x + \dfrac{8}{x}$;

Solution: $\boxed{8x^7 + 8^x \ln 8 - \dfrac{8}{x^2}\ (\textbf{5 points})}$

b) $(x^{201}+1)^{420}(x^{1/5} + \tan x + 1)$;

Solution:

$$\boxed{\Big(420(x^{201}+1)^{419}(201x^{200})\Big)\,(x^{1/5} + \tan x + 1) + (x^{201}+1)^{420}(\frac{1}{5}x^{-4/5} + \sec^2 x)\ (\textbf{5 points})}$$

c) $\tanh x + \ln(\cosh x) + \cos(x^5 + e^{3x})$;

Solution: $\boxed{\dfrac{1}{\cosh^2 x} + \dfrac{\sinh x}{\cosh x} - (\sin(x^5 + e^{3x}))(5x^4 + 3e^{3x})\ (\textbf{5 points})}$

d) $\dfrac{e^{\sinh x} + \sin^2 x - 5\pi}{x^3 - 12x^5}$.

Solution:

$$\boxed{\frac{(e^{\sinh x}\cosh x + 2\sin x\cos x)(x^3 - 12x^5) - (e^{\sinh x} + \sin^2 x - 5\pi)(3x^2 - 60x^4)}{(x^3 - 12x^5)^2}\ (\textbf{5 points})}$$

2. (15 points) Let $y = f(x)$ be a differentiable function which satisfies the equation

$$5x^4y^7 + 2x^7 = 3y^5.$$

a) Find $\frac{dy}{dx}$ in terms of x and y.

Solution: By implicit differentiation $(20x^3)y^7 + (5x^4)(7y^6y') + 14x^6 = 15y^4y'$. Therefore $\boxed{\frac{dy}{dx} = \frac{20x^3y^7 + 14x^2}{15y^4 - 35x^4y^6} \text{ (6 points)}}$

b) Find an equation of the line tangent to the graph of $y = f(x)$ at the point $(1, -1)$.

Solution: At the point $(1, -1)$ note that the slope of the tangent line is $\boxed{\frac{-20+14}{15-30} = \frac{3}{10} \text{ (3 points)}}$. Thus an equation for the tangent line is $\boxed{y + 1 = \frac{3}{10}(x-1) \text{ (3 points)}}$.

c) Use tangent line approximation to estimate $f(1.15)$.

Solution: $f(1.15) \approx (.3)(1.15 - 1) - 1 = .045 - 1 = \boxed{-.955 \text{ 3 points)}}$.

3. (18 points) Let $f(x) = x^4 - 2x^3$.

a) Find the *critical points* of $y = f(x)$.

Solution: $f'(x) = 4x^3 - 6x^2 = 2x^2(2x - 3)$. Thus the critical points are at $\boxed{x = 0, \frac{3}{2} \text{ (3 points)}}$.

b) Find the intervals on which $f(x)$ is *increasing*, on which $f(x)$ is *decreasing*.

Solution: Since $f'(-1) = -10 < 0, f'(1) = -2 < 0$, and $f'(2) = 8 > 0$,

$\boxed{f(x) \text{ is } \textit{increasing} \text{ on } (-\infty, 0], [0, \frac{3}{2}] \text{ (2 points)}}$ and

$\boxed{f(x) \text{ is } \textit{decreasing} \text{ on } [\frac{3}{2}, \infty) \text{ (1 point)}}$

c) Find the intervals on which the graph of $y = f(x)$ is *concave down*, on which the graph is *concave up*.

Solution: $f''(x) = 12x^2 - 12x = 12x(x-1)$. Since $f''(-1) = 24 > 0, f''(\frac{1}{2}) = -3 < 0$, and $f''(2) = 24 > 0$, the graph of $y = f(x)$ is $\boxed{\textit{concave up} \text{ on } (-\infty, 0), (1, \infty) \text{ (2 points)}}$ is $\boxed{\textit{concave down} \text{ on } (0, 1) \text{ (1 points)}}$.

d) Find the *inflection points* on the graph of $y = f(x)$.

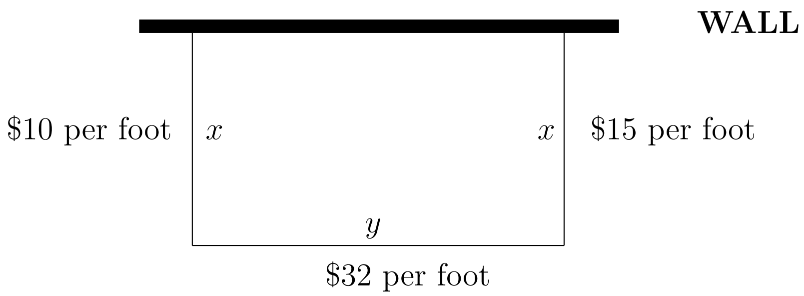
Solution: In light of part d) the inflection points are $\boxed{(0,0) \text{ and } (1,-1)\ (\textbf{3 points})}$.

e) Using the information derived from parts a)–d), sketch the graph of $y = f(x)$. Plot the points corresponding to local maxima and minima, inflection points, and points where the graph crosses the x-axis.

Solution: Since $f(x) = x^4 - 2x^3 = x^3(x-2)$ the graph of $y = f(x)$ crosses the x-axis at $x = 0, 2$. The point $(\frac{3}{2}, -\frac{27}{16})$ is a local minimum and there is no local maximum. (**3 points for shape of graph and 3 points for points**). See "Solution to Hour Exam II Problem 3" on our section's webpage for a sketch of the graph.

********* Over for Questions 4–6 *********

4. (17 points) A rectangular garden covering 200 square feet is constructed as follows: one side is against a straight wall to and (very expensive) plants are to border the other three sides with costs as indicated.



a) Find the total cost of the plants C in terms of the side length x and find the domain of $C = C(x)$.

Solution: $C(x) = 10x + 32y + 15x = 25x + 32\left(\frac{200}{x}\right)$ since $xy = 200$. Thus

$C(x) = 25x + \left(\frac{6400}{x}\right)$ (**6 points**). The domain of $C(x)$ is the set of all $x > 0$ (**1 point**).

b) Find the dimension (x and y) of the garden which *minimize* the total cost of the plants. *You must justify your answer.*

Solution: $C'(x) = 25 - \left(\frac{6400}{x^2}\right) = 0$, where $x > 0$, holds if and only if $x = 16$. Since $C'(1) < 0$ and $C'(25) > 0$, it follows that $C(x)$ has a minimum at $x = 16$. (**3 points for justification of minimum.**) The dimensions are therefore

$x = 16$ feet by $y = \frac{200}{16} = 12.5$ feet (**5 points**).

c) Find the minimal cost of the plants.

Solution: $C(16) = 25{\cdot}16 + \frac{6400}{16} = 800$ dollars (**2 points**).

5. (15 points) Use L'Hopital's rule to find the following limits:

a) $\lim\limits_{x\longrightarrow\infty} \frac{13x^2 + x - 6}{5x^2 + 4897005x + 492}$;

Solution:

$$\lim_{x\longrightarrow\infty} \frac{13x^2 + x - 6}{5x^2 + 4897005x + 492} = \lim_{x\longrightarrow\infty} \frac{26x + 1}{10x + 4897005} = \lim_{x\longrightarrow\infty} \frac{26}{10} = \frac{13}{5} \ (\textbf{7 points})$$

b) $\lim\limits_{z \longrightarrow 0} \dfrac{e^{19z} - 1}{e^{-14z} - 1}$.

Solution: $\boxed{\lim\limits_{z \longrightarrow 0} \dfrac{e^{19z} - 1}{e^{-14z} - 1} = \lim\limits_{z \longrightarrow 0} \dfrac{19e^{19z}}{-14e^{-14z}} = -\dfrac{19}{14} \text{ } (\textbf{8 points}).}$

6. (15 points) The surface area S of a sphere of a radius r is given by $S = 4\pi r^2$. If the Surface area is increasing at the rate of 7 square feet per minute, find the rate at which the radius is increasing when $r = 5$ feet.

Solution: $\boxed{\dfrac{dS}{dt} = 8\pi r \dfrac{dr}{dt} \text{ } (\textbf{5 points}).}$ Thus $\boxed{7 = (8\pi \cdot 5)\dfrac{dr}{dt} \text{ } (\textbf{5 points})}$ which means that $\boxed{\dfrac{dr}{dt} = \dfrac{7}{40\pi} \approx 0.0557 \text{ feet per minute } (\textbf{5 points})}$