MATH 180 **Quiz 10 (Version I) 04/13/06** **Solutions 04/15/06**

Name (print) ______________________ **Discussion hour (T Th** ______**)**

1. (10 pts.) Consider the following table for the velocity function f:

t	0	3	6	9	12	15
$f(t)$	6	11	13	16	24	36

where we assume that time t is in seconds and the velocity $f(t)$ is given in meters/sec.

- Find an *upper bound* and a *lower bound* for the distance traveled between $t = 0$ and $t = 15$.

  Solution : We have that $n = 5$ and therefore, $\Delta t = 3$. So, we compute the Left and Right sums:

  $$\begin{aligned} lhs &= (6 + 11 + 13 + 16 + 24) \cdot 3 = 210 \text{ meters} \\ rhs &= (11 + 13 + 16 + 24 + 36) \cdot 3 = 300 \text{ meters} \end{aligned}$$

  So, an *upper bound* is 300 meters, while a *lower bound* is 210 meters; in other words, if d is the distance covered between $t = 0$ and $t = 15$, we have that: $210 < d < 300$.

- Find the difference between the upper and lower bound *without* using your answer to the previous question. (You may of course use this answer to check your result!)

  Solution : We apply the formula $|U - L| = |f(a) - f(b)| \cdot \Delta t$ to get that:

  $$|U - L| = |f(0) - f(15)| \cdot 3 = |6 - 36| \cdot 3 = 30 \cdot 3 = 90 \text{ meters.}$$

  (Note that this agrees with the answer to the previous part.)

2. (10 pts.) With reference to the above function f, how frequently should we measure the velocity in order to estimate the total distance covered to within 4.5 meters ?

Solution : We want the difference between the upper and the lower estimate, to be less than or equal to 4.5 meters. This means that:

$$|U - L| \leq 4.5 \Rightarrow |f(0) - f(15)| \cdot \Delta t \leq 4.5 \Rightarrow 30 \cdot \Delta t \leq 4.5 \Rightarrow \Delta t \leq 0.15 \text{ secs.}$$

In particular, if we measure the velocity every 0.15 secs, the difference between the two bounds will be exactly 4.5 meters. (Note: If $\Delta t = 0.15$ secs, then $n = 100$.)

*** END OF VERSION I ***
*** OVER FOR VERSION 2 ***

MATH 180 **Quiz 10 (Version 2) 04/13/06** **Solutions 04/15/06**

Name (print) ______________________ **Discussion hour (T Th** ______**)**

1. (10 pts.) Consider the following table for the velocity function f:

t	0	3	6	9	12	15
$f(t)$	36	24	16	13	11	6

where we assume that time t is in seconds and the velocity $f(t)$ is given in meters/sec.

- Find an *upper bound* and a *lower bound* for the distance traveled between $t = 0$ and $t = 15$.

  Solution : We have that $n = 5$ and therefore, $\Delta t = 3$. So, we compute the Left and Right sums:

  $$\begin{aligned} lhs &= (36 + 24 + 16 + 13 + 11) \cdot 3 = 300 \text{ meters} \\ rhs &= (24 + 16 + 13 + 11 + 6) \cdot 3 = 210 \text{ meters} \end{aligned}$$

  So, an *upper bound* is 300 meters, while a *lower bound* is 210 meters; in other words, if d is the distance covered between $t = 0$ and $t = 15$, we have that: $210 < d < 300$.

- Find the difference between the upper and lower bound *without* using your answer to the previous question. (You may of course use this answer to check your result!)

  Solution : We apply the formula $|U - L| = |f(a) - f(b)| \cdot \Delta t$ to get that:

  $$|U - L| = |f(0) - f(15)| \cdot 3 = |36 - 6| \cdot 3 = 30 \cdot 3 = 90 \text{ meters.}$$

  (Note that this agrees with the answer to the previous part.)

2. (10 pts.) With reference to the above function f , how frequently should we measure the velocity in order to estimate the total distance covered to within 9 meters ?

Solution : We want the difference between the upper and the lower estimate, to be less than or equal to 9 meters. This means that:

$$|U - L| \leq 9 \Rightarrow |f(0) - f(15)| \cdot \Delta t \leq 9 \Rightarrow 30 \cdot \Delta t \leq 9 \Rightarrow \Delta t \leq 0.3 \text{ secs.}$$

In particular, if we measure the velocity every 0.3 secs, the difference between the two bounds will be exactly 9 meters. (Note: If $\Delta t = 0.3$ secs, then $n = 50$.)

*** END OF VERSION 2 ***