MATH 180 **Quiz 4 (Version I) 02/16/06** **Solution 02/17/06**

Name (print) ______________________ **Discussion hour (T Th** ____**)**

1. (10 pts.)

a) Compute the derivatives of the functions :

- $f(x) = 6\sqrt[3]{x} + \dfrac{4}{\sqrt[4]{x}}$

  Solution : We can write the function as $f(x) = 6\,x^{\frac{1}{3}} + 4\,x^{-\frac{1}{4}}$ and so, by using the power rule, we get:

  $$f'(x) = 6\left(x^{\frac{1}{3}}\right)' + 4\left(x^{-\frac{1}{4}}\right)' = 6\left(\frac{1}{3}\right)\,x^{-\frac{2}{3}} + 4\left(-\frac{1}{4}\right)\,x^{-\frac{5}{4}} = 2\,x^{-\frac{2}{3}} - x^{-\frac{5}{4}}.$$

  So answer is $\boxed{f'(x) = 2\,x^{-\frac{2}{3}} - x^{-\frac{5}{4}}}$ or $\boxed{f'(x) = \dfrac{2}{x^{2/3}} - \dfrac{1}{x^{5/4}}}$. (**3 pts**.)

  (Equivalent forms of final answer are acceptable and get full credit.)

- $g(x) = \dfrac{\sqrt{x}(1+2x)}{x^2}$

  Solution : We can write the function in the following way :

  $$g(x) = \frac{x^{\frac{1}{2}}(1+2x)}{x^2} = \frac{x^{\frac{1}{2}} + 2x^{\frac{3}{2}}}{x^2} = x^{\frac{1}{2}-2} + 2x^{\frac{3}{2}-2} = x^{-\frac{3}{2}} + 2x^{-\frac{1}{2}}.$$

  So, by using the power rule we get $\boxed{g'(x) = -\dfrac{3}{2}\,x^{-\frac{5}{2}} - x^{-\frac{3}{2}}}$. (**3 pts**.)

  (Again, equivalent forms of final answer are acceptable and get full credit.)

b) Find the equation of the line tangent to the graph of f at $x = 1$.

Solution : We are interested in the point $(1, f(1))$ i.e. $(1, 10)$. The slope at that point is $m = f'(1) = 1$ (using the formula of f' that we computed above). So the equation of the tangent line at that point is : $y - 10 = x - 1 \Rightarrow \boxed{y = x + 9}$. (**4 pts**.)

2. (10 pts.)

a) Compute the derivative of the function : $h(x) = 2\sqrt{x} - \left(\dfrac{1}{3}\right)^x$.

Solution : $h'(x) = 2\left(x^{\frac{1}{2}}\right)' - \left[\left(\dfrac{1}{3}\right)^x\right]' \Rightarrow \boxed{h'(x) = x^{-\frac{1}{2}} - \left(\ln\dfrac{1}{3}\right)\left(\dfrac{1}{3}\right)^x}$. (**5 pts**.)

b) Suppose that $P(t) = 50\,(1.2)^t$, expresses the growth of price P (in dollars) as a function of time t (in years). Find the rate, in dollars per year, at which the price P is increasing.

Solution : We actually want to find the derivative function P'. A direct differentiation gives:

$$P'(t) = 50\left[(1.2)^t\right]' \Rightarrow \boxed{P'(t) = 50(\ln 1.2)(1.2)^t \approx 9.12(1.2)^t}. \quad (\mathbf{5\ pts}.)$$

*** END OF VERSION I ***

*** OVER FOR VERSION 2 ***

MATH 180 **Quiz 4 (Version 2) 02/16/06** **Solution 02/17/06**

Name (print) ______________________________ **Discussion hour (T Th ______)**

1. (10 pts.)

   a) Compute the derivatives of the functions :

   - $f(x) = 8\sqrt[4]{x} + \dfrac{3}{\sqrt[3]{x}}$

     Solution : We can write the function as $f(x) = 8\,x^{\frac{1}{4}} + 3\,x^{-\frac{1}{3}}$ and so, by using the power rule, we get:

     $$f'(x) = 8\left(x^{\frac{1}{4}}\right)' + 3\left(x^{-\frac{1}{3}}\right)' = 8\left(\frac{1}{4}\right)x^{-\frac{3}{4}} + 3\left(-\frac{1}{3}\right)x^{-\frac{4}{3}} = 2\,x^{-\frac{3}{4}} - x^{-\frac{4}{3}}.$$

     So answer is $\boxed{f'(x) = 2\,x^{-\frac{3}{4}} - x^{-\frac{4}{3}}}$ or $\boxed{f'(x) = \dfrac{2}{x^{3/4}} - \dfrac{1}{x^{4/3}}}$. (**3 pts.**)

     (Equivalent forms of final answer are acceptable and get full credit.)

   - $g(x) = \dfrac{\sqrt[3]{x}(1+2x^{\frac{2}{3}})}{x^2}$

     Solution : We can write the function in the following way :

     $$g(x) = \frac{x^{\frac{1}{3}}(1+2x^{\frac{2}{3}})}{x^2} = \frac{x^{\frac{1}{3}} + 2x}{x^2} = x^{\frac{1}{3}-2} + 2x^{1-2} = x^{-\frac{5}{3}} + 2x^{-1}.$$

     So, by using the power rule we get $\boxed{g'(x) = -\dfrac{5}{3}\,x^{-\frac{8}{3}} - 2x^{-2}}$. (**3 pts.**)

     (Again, equivalent forms of final answer are acceptable and get full credit.)

   b) Find the equation of the line tangent to the graph of f at $x = 1$.

   Solution : We are interested in the point $(1, f(1))$ i.e. $(1, 11)$. The slope at that point is $m = f'(1) = 1$ (using the formula of f' that we computed above). So the equation of the tangent line at that point is : $y - 11 = x - 1 \Rightarrow \boxed{y = x + 10}$. (**4 pts.**)

2. (10 pts.)

   a) Compute the derivative of the function : $h(x) = 6\sqrt[3]{x} - \left(\dfrac{1}{2}\right)^x$.

   Solution : $h'(x) = 6\left(x^{\frac{1}{3}}\right)' - \left[\left(\dfrac{1}{2}\right)^x\right]' \Rightarrow \boxed{h'(x) = 2x^{-\frac{2}{3}} - \left(\ln\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)^x}$. (**5 pts.**)

b) Suppose that $P(t) = 60\,(1.3)^t$, expresses the growth of price P (in dollars) as a function of time t (in years). Find the rate, in dollars per year, at which the price P is increasing.

Solution : We actually want to find the derivative function P'. A direct differentiation gives:

$$P'(t) = 60\left[(1.3)^t\right]' \Rightarrow \boxed{P'(t) = 60(\ln 1.3)(1.3)^t \approx 15.74(1.3)^t}. \quad \textbf{(5 pts.)}$$

*** END OF VERSION 2 ***