MATH 180 (16397) Quiz 7 (Version I) 03/10/06; Solution 03/09/06

Name (print) ______________________ **Discussion hour (T Th __)**
Show your work. Answers alone are *not* sufficient.

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1. (8 pts.) Let $f(x) = x^3 - 3x^2 + 3ax - 17$, where a is a fixed real number.

(a) Find the *number* of critical points of $f(x)$ when $a = 1$.

Solution: $f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1) = 3(x-1)^2$ so $\boxed{f'(x) = 3(x-1)^2 \ (\textbf{2 points})}$. Therefore $f(x)$ has $\boxed{\text{one critical point } (\textbf{2 points})}$, at $x = 1$.

(b) For any fixed value of a, find all of the inflection points on the graph of $y = f(x)$.

Solution: $\boxed{f''(x) = 6x - 6 = 6(x-1) \ (\textbf{1 point}).}$ Since the concavity goes from down to up as x passes through $x = 1$, there is one inflection point $\boxed{(1, 3a - 19) \ (\textbf{3 points})}$.

2. (12 pts.) Let $f(x) = 16x + \frac{9}{x}$, where $x > 0$.

(a) Find the intervals on which $f(x)$ is *increasing.*

Solution: $f'(x) = 16 - \frac{9}{x^2} = \frac{16x^2 - 9}{x^2}$ (**2 points**) which means there is only one critical point which is at $x = \frac{3}{4}$. Since $f'(\frac{1}{2}) < 0$ and $f'(1) > 0$, the function $f(x)$ is increasing on $\boxed{[\frac{3}{4}, \infty). \ (\textbf{3 points})}$

(b) Find the intervals on which $f(x)$ is *decreasing.*

Solution: Using the calculations of part (a) we see that the function is decreasing on $\boxed{(0, \frac{3}{4}]. \ (\textbf{3 points})}$

(c) Find all *local minima* for $f(x)$. *Justify your answer by the second derivative test.*

Solution: There is only one critical point at $x = \frac{3}{4}$. Since $f''(x) = \frac{18}{x^3}$ it follows that

$\boxed{f''(\frac{3}{4}) > 0 \text{ and thus } (\frac{3}{4}, 24) \text{ is a local minumum. } (\textbf{4 points})}$

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1. (8 pts.) Let $f(x) = x^3 + 6x^2 + 3ax + 5$, where a is a fixed real number.

   (a) Find the *number* of critical points of $f(x)$ when $a = 0$.

   Solution: $f'(x) = 3x^2 + 12x = 3x(x+4)$ so $\boxed{f'(x) = 3x(x+4)\ (\textbf{2 points}).}$ Therefore $f(x)$ has $\boxed{\text{two critical points } (\textbf{2 points})}$, at $x = 0, -4$.

   (b) For any fixed value of a, find all of the inflection points on the graph of $y = f(x)$.

   Solution: $\boxed{f''(x) = 6x + 12 = 6(x+2)\ (\textbf{1 point}).}$ Since the concavity goes from down to up as x passes through $x = -2$, there is one inflection point $\boxed{(-2, 21 - 6a)\ (\textbf{3 points})}$.

2. (12 pts.) Let $f(x) = 25x + \frac{4}{x}$, where $x > 0$.

   (a) Find the intervals on which $f(x)$ is *increasing*.

   Solution: $f'(x) = 25 - \frac{4}{x^2} = \frac{25x^2 - 4}{x^2}$ (**2 points**) which means there is only one critical point which is at $x = \frac{2}{5}$. Since $f'(\frac{1}{5}) < 0$ and $f'(1) > 0$, the function $f(x)$ is increasing on $\boxed{[\frac{2}{5}, \infty).\ (\textbf{3 points})}$

   (b) Find the intervals on which $f(x)$ is *decreasing*.

   Solution: Using the calculations of part (a) we see that the function is decreasing on $\boxed{(0, \frac{2}{5}].\ (\textbf{3 points})}$

   (c) Find all *local minima* for $f(x)$. *Justify your answer by the second derivative test.*

   Solution: There is only one critical point at $x = \frac{2}{5}$. Since $f''(x) = \frac{8}{x^3}$ it follows that $\boxed{f''(\frac{2}{5}) > 0 \text{ and thus } (\frac{2}{5}, 20) \text{ is a local minumum. } (\textbf{4 points})}$