MATH 431 **Spring 2009** **Radford**

Written Homework #12 (Revised)

Due at the beginning of class 04/29/2009

The problems are from our text. You may use the following:

(A) Suppose that E is a field extension of F and $f(x) \in F[x]$. Then $\sigma \in \mathrm{Gal}(E/F)$ permutes the roots of $f(x)$ in E.

(B) Suppose that E is a field extension of F and $E = F(S)$. If $\sigma, \tau \in \mathrm{Gal}(E/F)$ and $\sigma(s) = \tau(s)$ for all $s \in S$ then $\sigma = \tau$.

(C) Let $p, q \in \mathbf{Z}$ be distinct primes (positive or negative). Then $[\mathbf{Q}(\sqrt{p}, \sqrt{q}) : \mathbf{Q}] = 4$.

(D) Suppose E is a field extension of F and $a, b \in E$ satisfy $[F(a) : F], [F(b) : F]$ are finite and are relatively prime. Then $[F(a, b) : F]$ is finite and $[F(a, b) : F] = [F(a) : F][F(b) : F]$.

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1. Page 560, number 4.

2. Page 560, number 10.

3. Page 561, number 12. Find generators and relations for the Galois group of $x^3 - 2$ over $\mathbf{Q}$.

4. Page 561, number 16.