

Name (print) _____ Tu/Th Discussion (circle) 8 9 10

(1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *Show* your work. (4) There are *six questions* on this exam. (5) If you use a calculator it must be *your own*. (6) Round decimal answers to *three decimal places*. (7) *You are expected to abide by the University's rules concerning academic honesty.*

1. (20 points) The population $P(t)$ of a small town is growing exponentially. Time t is measured in years. Given that $P(5) = 2,000$ and $P(6) = 2,600$, find:

- a) $P(7)$;
- b) $P(0)$;
- c) the annual growth rate;
- d) the continuous growth rate.

2. (15 points) Let $f(x) = 3x^2 + 7x$. Starting with the difference quotient, use algebra to find $f'(x)$.

3. (18 points) Sketch the graph of a function which has *all* of the following properties:

- a) $\lim_{x \rightarrow 0} f(x) = \infty$, $\lim_{x \rightarrow 9^-} f(x) = \infty$, and $\lim_{x \rightarrow 9^+} f(x) = -\infty$;
- b) $f'(x) > 0$ on the intervals $(-\infty, 0)$, $(5, 9)$, and $(9, \infty)$;
- c) $f'(x) < 0$ on the interval $(0, 5)$;
- d) $f''(x) > 0$ on the intervals $(0, 2)$ and $(4, 5)$;
- e) $f''(x) < 0$ on the interval $(2, 4)$.

You must label the numbers 0, 2, 4, 5 and 9 on your x -axis.

4. (17 points) Let $f(x) = x^3 + 2/x$. Then $f'(x) = 3x^2 - 2/x^2$.

- a) Find an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$.
- b) Use tangent line approximation to estimate the value of $f(2.25)$.

*** OVER FOR PROBLEMS 5 AND 6 ***

5. (15 points) Let $f(x) = \begin{cases} 4x^3 + 5x - 1 & : x \leq 1 \\ 3x^5 + 7x^4 - x^2 & : x > 1 \end{cases}$.

- a) What is $\lim_{x \rightarrow 1^-} f(x)$?
- b) What is $\lim_{x \rightarrow 1^+} f(x)$?
- c) Is $f(x)$ continuous at $x = 1$? Explain.

6. (15 points) Suppose that $f(x)$ is defined and $f'(x)$ exists for all real numbers x . Below is a table of values of the function.

x	-1	1	4
$f(x)$	3	7	11

- a) Use the table above to calculate the average rate of change of $f(x)$ on the intervals $[-1, 1]$ and $[1, 4]$.
- b) Estimate the derivative $f'(x)$ at $x = -1, 1, 4$. (Use averaging to estimate $f'(1)$.)
- c) Is the statement “The graph of $y = f(x)$ is concave up on the interval $(-2, 5)$ ” consistent with the derivative estimates of part b)? Explain.