MATH 180 Hour Exam I Solution **Radford** **02/18/02**

1. (20 points) a) $P(t) = P_0 a^t = P(0)a^t$. The base $a = P(6)/P(5) = 2600/2000 = 1.3$. Therefore $P(7) = P(6)a = (2600)(1.3) = \mathbf{3,380}$. (5 pts.) b) $2000 = P(5) = P(0)(1.3)^5$. Therefore $P(0) = 2000/(1.3)^5 \doteq \mathbf{538}$. (5 pts.) c) $a = 1.3 = 1 + .3$; thus the annual growth rate is $.\mathbf{3}$ or $\mathbf{30}\%$. (5 pts.) d) $1.3 = a = e^k$ means that $k = \ln(1.3) \doteq .262364$. Therefore the continuous growth rate is $.\mathbf{262}$ or $\mathbf{26.236}\%$ to three decimal places.

2. (15 points) We first evaluate the difference quotient at $x = a$

$$\begin{aligned}\frac{f(a+h)-f(a)}{h} &= \frac{[3(a+h)^2+7(a+h)]-[3a^2+7a]}{h}\\ &= \frac{[3(a^2+2ah+h^2)+7(a+h)]-[3a^2+7a]}{h}\\ &= \frac{3(2ah+h^2)+7(h)}{h}\\ &= \frac{h(3(2a+h)+7)}{h}\\ &= 3(2a+h)+7 \quad \text{(10 pts.)}\end{aligned}$$

and then compute

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} 3(2a+h)+7 = 6a+7. \quad \text{(3 pts.)}$$

Therefore $f'(x) = 6x + 7$. (2 pts.)

3. (18 points) Vertical asymtotes (6 pts.), intervals where increasing, decreasing (8 pts.), and intervals of concavity within the interval $(0, 5)$ (4 pts.)

4. (17 points) a) $f(2) = 2^3 + 2/2 = \mathbf{9}$ and $f'(2) = 3(2^2) - 1/2 = \mathbf{23/2}$ (4 pts.) $\mathbf{y - 9 = (23/2)(x - 2)}$ (6 pts.) b) The estimate is $y = (23/2)(2.25 - 2) + 9 = (23/8) + 9 = 95/8$; thus our estimate for $f(2.25)$ is $\mathbf{95/8}$ or 11.875 (7 pts.)

5. (15 points) a) Using continuity $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 4x^3+5x-1 = 4(1)^3+5(1)-1 = \mathbf{8}$. (4 pts.) b) Using continuity $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3x^5 + 7x^4 - x^2 = 3(1)^5 + 7(1)^4 - 1 = \mathbf{9}$. (4 pts.) c) $f(x)$ is *not* continuous at $x = 1$. Since $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$ are not the same, $\lim_{x\to 1} f(x)$ does not exist and therefore $f(x)$ is not continuous at $x = 1$. (7 pts.)

6. (15 points) a) $(f(1) - f(-1))/(1 - (-1)) = (7 - 3)/2 = \mathbf{2}$ and $(f(4) - f(1))/(4 - 1) = 1 - 7)/3 = \mathbf{4/3}$ (5 pts.) b) $f'(-1) \doteq \mathbf{2}$, $f'(1) \doteq (1/2)(2 + 4/3) = \mathbf{5/3}$, and $f'(4) \doteq \mathbf{4/3}$ (5 pts.). Tabulating:

x	-1	1	4
approximation of $f'(x)$	2	5/3	4/3

c) *Not* consistent. The derivative estimates indicate that $f'(x)$ is not increasing, that is $f''(x) = (f')'(x)$ is not positive throughout the interval. (5 pts.)