MATH 180 **Hour Exam I Review** **Radford** **02/11/02**

Here are some review problems which supplement Sample Hour Exam 1, attributed to Professor S. Smith, found under "Samples" on the Math180 homepage. *These problems, together with these sample hour exam I problems, are NOT meant to be exhaustive.*

1. Let $f(x) = (3x^2 + 4x - 1)/(4x + 3)$

   a) Estimate $\lim_{x \to 1} f(x)$ by calculating $f(x)$ for $x = 0.5,\ 0.6,\ 0.7,\ 0.9,\ 0.99,\ 0.999,\ 0.9999$, and $x = 1.1,\ 1.01,\ 1.001,\ 1.0001$.

   b) Use the rules for computing limits described in Theorem 2.1 to calculate $\lim_{x \to 1} f(x)$.

   c) Assuming that $f(x)$ is continuous at $x = 1$, calculate $\lim_{x \to 1} f(x)$.

2. Let $f(x) = \begin{cases} 3x^2 + 2x^3 + 7 & : & x < 0 \\ 6x^4 + x^3 + 2 & : & x \geq 0 \end{cases}$

   a) Find $\lim_{x \to 0} 3x^2 + 2x^3 + 7$.

   b) Find $\lim_{x \to 0} 6x^4 + x^3 + 2$.

   c) Find $\lim_{x \to 0^-} f(x)$.

   d) Find $\lim_{x \to 0^+} f(x)$.

   e) Find all a such that $f(x)$ is continuous at $x = a$.

   f) Find all a such that $f'(a)$ exists. (You may assume that polynomials are differentiable everywhere.)

3. Suppose that $f'(x) = x^2 - 6x + 8$.

   a) Starting with the definition of derivative, use algebra to find $f''(x)$.

   b) Describe all intervals on which the graph of $y = f(x)$ is concave down.

   c) Describe all intervals on which the $f'(x)$ is increasing.

   d) Describe all intervals on which the graph of $y = f(x)$ is concave up.

   e) Describe all intervals on which the $f(x)$ is increasing.

4. Let $s(t) = t^3 - 4t^2 - 3t$ describe the position of a body moving along a straight line at time $t \geq 0$.

   a) Given that the instantaneous velocity of the body at time t is $v(t) = s'(t) = 3t^2 - 8t - 3$, find the acceleration $a(t)$ of the body at time t.

b) When is the body moving to the right? To the left?

c) When is the velocity of the body increasing? Decreasing?

d) At what times is the body at the point where the motion originates?

5. Suppose that $f(x)$ is defined on all real numbers and $f'(x)$ exists for all real numbers x. Below is a table of values of the function.

x	-1	0	2	5	7
f(x)	3	4.5	5.4	6.0	6.2

(1)

a) Use the table above to complete the table below which records average rates of change of $f(x)$ on the interval $[-1, 0]$, $[0, 2]$, ...

interval	-1 – 0	0 – 2	2–5	5–7
average rate of change				

b) Use the table (1) to complete the table below by estimating the instantaneous rate of change of $f(x)$ at the indicated values of x.

x	-1	0	2	5	7
f′(x)					

c) Use the table constructed in part b) to complete the table below by estimating the second derivative.

x	-1	0	2	5	7
f″(x)					

6. Let $f(x) = x + 1/(1 + x^2)$. Then $f'(x) = 1 - (2x/(1 + x^2)^2)$.

a) Find the average rate of change of $f(x)$ on the interval $[-1, 2]$.

b) Find the instantaneous rate of change of $f(x)$ at $x = -3$.

c) Find the derivative of $f(x)$ at $x = 7$.

d) Find the slope of the line tangent to the graph of $y = f(x)$ at the point $(1, 3/2)$.

e) Find an equation of the tangent line to the graph of $y = f(x)$ at $x = 4$.

f) Use tangent line approximation to estimate $f(4.03)$.

g) Assume that $f(x)$ is the cost in dollars of producing x items, $x > 0$. Use the derivative to estimate the cost of producing item number 101.

7. Sample Hour Exam I (S. Smith's version), problem 2). Also determine a) the continuous growth rate, b) the annual growth rate, and c) the time it takes for the investment to double.