

Name (print) \_\_\_\_\_ Tu/Th Discussion (circle) 12 1 2

(1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *Show* your work. (4) There are *six questions* on this exam. (5) If you use a calculator it must be *your own*. (6) Round decimal answers to *four decimal places*. (7) *You are expected to abide by the University's rules concerning academic honesty.*

1. (20 points) The population  $P(t)$ , where  $t$  is years, of a city is growing exponentially at the rate of 18% per year. Given that  $P(4) = 2,000$  find (a)  $P(t)$ , (b)  $P(5)$ , and (c) the continuous growth rate.

2. (15 points) Let  $y = f(x) = 4x^2 + 5x$ . Starting with the difference quotient, use algebra to calculate  $f'(x)$ .

3. (18 points) Sketch the graph of a function  $y = f(x)$  which has *all* of the following properties:

(a)  $\lim_{x \rightarrow 0} f(x) = -\infty$ ,  $\lim_{x \rightarrow 3^-} f(x) = \infty$ ,  $\lim_{x \rightarrow 3^+} f(x) = -\infty$ ,  $\lim_{x \rightarrow -\infty} f(x) = 0$ ,  $\lim_{x \rightarrow \infty} f(x) = 0$ ;

(b)  $f'(x) > 0$  on the intervals  $(-\infty, -2)$ ,  $(0, 3)$ , and  $(3, \infty)$ ;

(c)  $f'(x) < 0$  on the interval  $(-2, 0)$ ;

(d)  $f''(x) > 0$  on the intervals  $(-\infty, -3)$  and  $(1, 3)$ ;

(e)  $f''(x) < 0$  on the interval  $(-3, 0)$ ,  $(0, 1)$ , and  $(3, \infty)$ .

**You must label the numbers  $-3, -2, -1, 0, 1, 3$  on your  $x$ -axis.**

4. (17 points) Let  $f(x) = 5x + \frac{4}{x+1}$ . Then  $f'(x) = 5 - \frac{4}{(x+1)^2}$ .

(a) Find an equation to the line tangent to the graph of  $y = f(x)$  at  $x = 9$ .

(b) Assume that  $f(x)$  is the total cost of producing  $x$  items. Using the derivative, *approximate* the cost of producing the 11<sup>th</sup> item.

5. (15 points) Let  $f(x) = \begin{cases} x^3 - 4x^2 + 17 & : x \leq 2 \\ 3x + 5 & : x > 2 \end{cases}$ . Compute (a)  $\lim_{x \rightarrow 2^-} f(x)$ ,

(b)  $\lim_{x \rightarrow 2^+} f(x)$ , and (c) determine whether or not  $y = f(x)$  is continuous at  $x = 2$  from your limit calculations.

6. (15 points) A particle moving along a straight line has position  $s(t) = 2t^3 - 3t^2 - 12t + 1$  at time  $t$ , where  $t \geq 0$ . Given that  $s'(t) = 6t^2 - 6t - 12$  and  $s''(t) = 12t - 6$ , find (a) when the particle is moving to the right, (b) to the left, and (c) when its velocity is increasing.