MATH 180 # Hour Exam I Solution **Radford 10/08/04**

1. (20 points total) a) $P(t) = P(0)a^t$, where $\boxed{a = 1 + .18 = 1.18 \quad \text{(5 points)}.}$ To find $P(0)$ we note $2000 = P(4) = P(0)(1.18)^4$. Thus $P(0) = \dfrac{2000}{1.18^4} \approx \boxed{031.5778. \text{ (5 points)}.}$ Combining we have $P(t) = 1031.5778(1.18)^t$, which could be expressed $P(t) = 1031.5778e^{(\ln 1.18)t}$. b) $P(5) = P(4)(1.18) = (2000)(1.18)$ $\boxed{= 2360 \text{ (5 points)}}$. c) $r = \ln 1.18 \approx$ $\boxed{0.1655, \text{ or } 16.55\% \text{ (5 points)}}$.

2. (15 points total)

$$\begin{aligned}
f'(x) &= \boxed{\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \text{ (5 points)}} \\
&= \lim_{h\to 0} \frac{[4(x+h)^2 + 5(x+h)] - [4x^2 + 5x]}{h} \\
&= \lim_{h\to 0} \frac{[4(x^2 + 2xh + h^2) + 5(x+h)] - [4x^2 + 5x]}{h} \\
&= \lim_{h\to 0} \frac{4(2xh + h^2) + 5h}{h} \\
&= \lim_{h\to 0} \frac{h(8x + 4h + 5)}{h} \\
&= \boxed{\lim_{h\to 0} 8x + 4h + 5 \text{ (5 points for calculations to this point)}} \\
&= \boxed{8x + 5. \text{ (5 points)}}
\end{aligned}$$

3. (18 points total) $\boxed{\text{Asymtoptes 6 points, increase/decrease 6 points, concave up/down 6 points.}}$

4. (17 points total) a) $f'(0) = 5 - \dfrac{4}{(9+1)^2} = \boxed{\dfrac{496}{100} \text{ or } 4.96 \text{ (6 points)}}$ and $f(9) = \dfrac{454}{100} = 45.4$. Thus $\boxed{y - 45.4 = 4.96(x - 9) \text{ or } y = 4.96x + .76 \text{ (6 points)}.}$ b) $f'(10) = 5 - \dfrac{5}{(10+1)^2} =$ $\boxed{\dfrac{496}{121} \text{ or } 4.9669. \text{ (5 points)}}$

5. (15 points total) a) $\lim_{x\longrightarrow 2^-} f(x) = \lim_{x\longrightarrow 2^-}(x^3 - 4x^2 + 17) = 8 - 16 + 17 =$ $\boxed{9 \text{ (5 points)}}$ and b) $\lim_{x\longrightarrow 2^+} f(x) = \lim_{x\longrightarrow 2^+}(3x + 5) = 6 + 5 = \boxed{11 \text{ (5 points)}.}$ c) Since $\lim_{x\longrightarrow 2^-} f(x) \neq \lim_{x\longrightarrow 2^+} f(x)$ we conclude that $\lim_{x\longrightarrow 2} f(x)$ does not exist. Therefore $\boxed{f(x) \text{ is not continuous at } x = 2 \text{ (5 points; explanation involving limits needed)}.}$

6. (15 points total) a) The particle is moving to the right when its velocity $s'(t)$ is positive, $t \geq 0$. Since $s'(t) = 6(t-2)(t+1)$ $\boxed{t > 2 \text{ (5 points)}.}$ b) The particle is moving to the left

when its velocity $s'(t)$ is negative, $t \geq 0$. Since $s'(t) = 6(t-2)(t+1)$ $\boxed{0 < t < 2 \text{ (5 points).}}$

c) The velocity $s'(t)$ is increasing when *its* derivative $s''(t) = 12t - 6 = 12(t - \frac{1}{2})$ is positive, $t \geq 0$. $\boxed{t > \frac{1}{2} \text{ (5 points).}}$

Comment: Precise formulations of where a function is increasing or where a function is decreasing, which we have not had yet, would give answers $t \geq 2$, $0 \leq t \leq 2$, $t \geq \frac{1}{2}$ to parts a), b), and c), respectively.