Name (print) ————————————————— Tu/Th Discussion (circle)   12   1   2

(1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *Show* your work. (4) There are **SIX QUESTIONS** on this exam. (5) If you use a calculator it must be *your own*. (6) Round decimal answers to *four decimal places*. (7) *You are expected to abide by the University's rules concerning academic honesty.*

1. (20 points) Find the derivatives of the following functions; *do NOT simplify answers.*

   a) $x^7 + 7^x + \dfrac{3}{\sqrt{x}}$;

   b) $(x^{20} + 1)^{30}(x^{1/3} + \cos x + 1)$;

   c) $\sinh x + \ln(\tan x) + \sin(x^2)$;

   d) $\dfrac{e^{\tanh x} + x^9 - 5}{4x^3 - 11x^5}$.

2. (15 points) Let $y = f(x)$ be a differentiable function which satisfies the equation

   $$x^3 y^2 + 4y^5 + x^3 = 20y.$$

   a) Find $\dfrac{dy}{dx}$ in terms of x and y.

   b) Find an equation of the line tangent to the graph of $y = f(x)$ at the point $(2, 1)$.

   c) Use tangent line approximation to estimate $f(2.25)$.

3. (18 points) Let $f(x) = 6x^2 - x^4$.

   a) Find the *critical points* of $y = f(x)$.

   b) Find the intervals on which $f(x)$ is *increasing*, on which $f(x)$ is *decreasing*.

   c) Find the intervals on which the graph of $y = f(x)$ is *concave down*, on which the graph is *concave up*.

   d) Find the *inflection points* on the graph of $y = f(x)$.

   e) Using the information derived from parts a)–d), sketch the graph of $y = f(x)$. Plot the points corresponding to local maxima and minima, inflection points, and points where the graph crosses the x-axis.

********* **OVER for Questions 4–6** *********

4. (17 points) A rectangular garden covering 1200 square feet is to be fenced. The cost of fencing for each side is indicated in the diagram below.

$15 per foot

$7 per foot y $\qquad$ y $5 per foot

x

x

$10 per foot
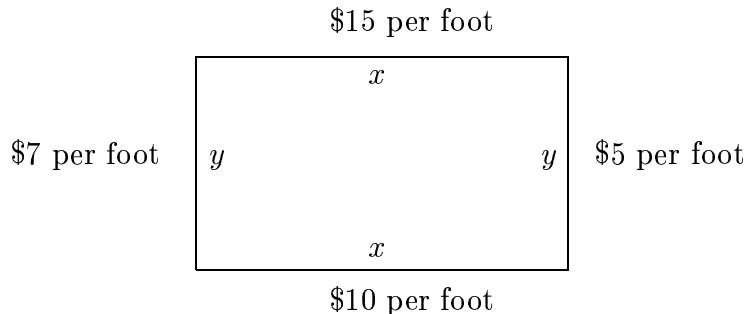
a) Find the total cost of the fencing C in terms of the side length x and find the domain of $C = C(x)$.

b) Find the dimension (x and y) of the garden which *minimize* the total cost of the fencing. You must justify your answer.

c) Find the minimal cost of the fencing.

5. (15 points) Use L'Hopital's rule to find the following limits:

a) $\lim\limits_{x \to \infty} \dfrac{8x^2 + 13x - 6}{11x^2 - 5x + 4}$;

b) $\lim\limits_{x \to 0} \dfrac{e^{3x} - 1}{7x}$.

6. (15 points) The volume V of a sphere of radius r is given by $V = \dfrac{4}{3}\pi r^3$. If the volume is increasing at the rate of 6 cubic feet per minute, find the rate at which the radius is increasing when $r = 4$ feet.