MATH 180 Quiz 6 (Version I) Solution **Radford** **12/02/04**

1. (12 pts.) Given the following table of values for the function $y = f(x)$

x	0	.5	1	1.5	2	2.5	3
$f(x)$	2	-1	4	0	-11	12	-8

find:

a) The left hand sum, where $a = 0$, $b = 3$, and $n = 3$;

Solution: $\Delta x = \dfrac{3-0}{3} = 1$. Thus the left hand sum is

$$\boxed{f(0)(1) + f(1)(1) + f(2)(1) = 2 + 4 - 11 \ \ (\mathbf{3\ \ points})} = \boxed{-5.\ \ (\mathbf{3\ \ points})}$$

b) The right hand sum, where $a = .5$, $b = 2.5$, and $n = 4$.

Solution: $\Delta x = \dfrac{2.5 - .5}{4} = .5$. Thus the right hand sum is

$$\boxed{f(1)(.5) + f(1.5)(.5) + f(2)(.5) + f(2.5)(.5) = (4 + 0 - 11 + 12)(.5) \ \ (\mathbf{3\ \ points})} =$$
$$\boxed{2.5.\ \ (\mathbf{3\ \ points})}$$

2. (8 pts.) Use the Fundamental Theorem of Calculus to find the average value of $y = f(x) = x^2 + 3x$ on the closed interval $[-1, 2]$.

Solution: Let $\boxed{F(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 \ (\mathbf{2\ points})}$. Then $F'(x) = f(x)$. Thus the average value is

$$\boxed{\frac{1}{2-(-1)}\int_{-1}^{2} F'(x)\,dx = \frac{F(2) - F(-1)}{3} \quad (\mathbf{3\ \ points})} = \frac{(\frac{8}{3} + 6) - (\frac{-1}{3} + \frac{3}{2})}{3} = \boxed{\frac{5}{2} \ \ (\mathbf{3\ \ points})}.$$

MATH 180 # Quiz 6 (Version II) Solution **Radford**

12/02/04

1. (12 pts.) Given the following table of values for the function $y = f(x)$

x	0	.5	1	1.5	2	2.5	3
$f(x)$	2	-1	4	0	-11	12	-8

find:

a) The right hand sum, where $a = 0$, $b = 3$, and $n = 3$;

Solution: $\Delta x = \dfrac{3-0}{3} = 1$. Thus the right hand sum is

$$\boxed{f(1)(1) + f(2)(1) + f(3)(1) = 4 - 11 - 8 \ \ (\mathbf{3\ \ points})} = \boxed{-15 \ \ (\mathbf{3\ \ points})}.$$

b) The left hand sum, where $a = .5$, $b = 2.5$, and $n = 4$.

Solution: $\Delta x = \dfrac{2.5 - .5}{4} = .5$. Thus the left hand sum is

$$\boxed{f(.5)(.5) + f(1)(.5) + f(1.5)(.5) + f(2)(.5) = (-1 + 4 + 0 - 11)(.5) \ \ (\mathbf{3\ \ points})} =$$

$$\boxed{-4. \ (\mathbf{3\ \ points})}$$

2. (8 pts.) Use the Fundamental Theorem of Calculus to find the average value of $y = f(x) = x^2 + 4x$ on the closed interval $[-2, 1]$.

Solution: Let $\boxed{F(x) = \frac{1}{3}x^3 + 2x^2 \ \ (\mathbf{2\ \ points})}$. Then $F'(x) = f(x)$. Thus the average value is

$$\boxed{\frac{1}{1-(-2)} \int_{-2}^{1} F'(x)\, dx = \frac{F(1) - F(-2)}{3} \ \ \ (\mathbf{3\ \ points})} = \frac{(\frac{1}{3} + 2) - (\frac{-8}{3} + 8)}{3} = \boxed{-1 \ \ (\mathbf{3\ \ points})}.$$