MATH 180 Quiz 4 (Version I) Solution **Radford** **11/09/04**

1. (10 pts.) Let $f(x) = \frac{x}{9} + \frac{1}{x}$, where $x > 0$.

   a) Find the critical points of $y = f(x)$ in the closed interval $[1,5]$.

   Solution: $f'(x) = \frac{1}{9} - \frac{1}{x^2} = 0$ has solutions $x = -3, 3$. $\boxed{x = 3}$ (**4 points**)

   b) Find the *maximum* value of $y = f(x)$ on the closed interval $[1,5]$.

   Solution: The maximum and miniumum are among $f(1) = \frac{10}{9}$, $f(3) = \frac{2}{3}$, $f(5) = \frac{34}{45}$. Thus the maximum is $\boxed{\frac{10}{9}}$ (**3 points**)

   c) Find the *minimum* value of $y = f(x)$ on the closed interval $[1,5]$.

   Solution: From the calculations in part b) the minimum is $\boxed{\frac{2}{3}}$ (**3 points**)

2. (10 pts.) Given that the *derivative* of $y = f(x)$ is $f'(x) = x^2 - 3x + 2$:

   a) Find the critical points of $y = f(x)$.

   Solution: $f'(x) = (x-1)(x-2) = 0$ has solutions $x = 1, 2$. $\boxed{x = 1, 2}$ (**2 points**)

   b) Find the values of x where a *local maximum* of $y = f(x)$ occurs.

   Solution: Since the solutions to $f'(x) = 0$ are $x = 1$ and $x = 2$, $f'(0) > 0$, $f'(1.5) < 0$, and $f'(3) > 0$, we have that $y = f(x)$ is increasing on $(-\infty, 1]$ and $[2, \infty)$ and is decreasing on $[1,2]$. Thus a local maximum occurs at $\boxed{x = 1.}$ (**2 points**)

   c) Find the values of x where a *local minimum* of $y = f(x)$ occurs.

   Solution: From the comments in part b) a local minumum occurs at $\boxed{x = 2.}$ (**2 points**)

   d) Find the intervals on which $f(x)$ is *increasing*.

   Solution: From the comments in part b) $\boxed{(-\infty, 1] \cup [2, \infty).}$ (**4 points**)

MATH 180 **Quiz 4 (Version II) Solution** **Radford**

11/09/04

1. (10 pts.) Let $f(x) = \frac{x}{4} + \frac{1}{x}$, where $x > 0$.

   a) Find the critical points of $y = f(x)$ in the closed interval $[1, 6]$.

   Solution: $f'(x) = \frac{1}{4} - \frac{1}{x^2} = 0$ has solutions $x = -2, 2$. $\boxed{x = 2}$ (**4 points**)

   b) Find the *maximum* value of $y = f(x)$ on the closed interval $[1, 6]$.

   Solution: The maximum and miniumum are among $f(1) = \frac{5}{4}, f(2) = 1, f(6) = \frac{5}{3}$. Thus the maximum is $\boxed{\frac{5}{3}}$ (**3 points**)

   c) Find the *minimum* value of $y = f(x)$ on the closed interval $[1, 6]$.

   Solution: From the calculations in part b) the minimum is $\boxed{1}$ (**3 points**)

2. (10 pts.) Given that the *derivative* of $y = f(x)$ is $f'(x) = x^2 - 5x + 6$:

   a) Find the critical points of $y = f(x)$.

   Solution: $f'(x) = (x - 2)(x - 3) = 0$ has solutions $x = 2, 3$. $\boxed{x = 2, 3}$ (**2 points**)

   b) Find the values of x where a *local maximum* of $y = f(x)$ occurs.

   Solution: Since the solutions to $f'(x) = 0$ are $x = 2$ and $x = 3$, $f'(0) > 0$, $f'(2.5) < 0$, and $f'(4) > 0$, we have that $y = f(x)$ is increasing on $(-\infty, 2]$ and $[3, \infty)$ and is decreasing on $[2, 3]$. Thus a local maximum occurs at $\boxed{x = 2.}$ (**2 points**)

   c) Find the values of x where a *local minimum* of $y = f(x)$ occurs.

   Solution: From the comments in part b) a local minumum occurs at $\boxed{x = 3.}$ (**2 points**)

   d) Find the intervals on which $f(x)$ is *increasing*.

   Solution: From the comments in part b) $\boxed{(-\infty, 2] \cup [3, \infty).}$ (**4 points**)