MATH 180 Quiz 3 (Version I) Solution **Radford** **10/22/04**

1. (12 points) Let $y = f(x)$ be defined implicitly by the equation $2x^2y + y^3 + x = 11$.

a) Find $\dfrac{dy}{dx}$ or y'.

Solution: By implicit differentiation $4xy + 2x^2y' + 3y^2y' + 1 = 0$. Therefore $(2x^2 + 3y^2)y' = -(1 + 4xy)$ and $\boxed{\dfrac{dy}{dx} = y' = -\dfrac{1 + 4xy}{2x^2 + 3y^2}.}$ (**6 points**)

b) Find an equation of the line tangent to the graph of $y = f(x)$ at the point $(-1, 2)$.

Solution: By part a) $y'(-1) = -\dfrac{1 + 4(-1)2}{2(-1)^2 + 3(2)^2} = -\dfrac{-7}{2 + 12} = \dfrac{1}{2}$. Therefore

$\boxed{y - 2 = \dfrac{1}{2}(x + 1) \text{ or } y = \dfrac{1}{2}x + \dfrac{5}{2}}$ (**6 points**)

2. (8 points) A particle is moving along a curve in the plane which is parameterized by $x = t^3 - t - 1$ and $y = t^4 + 2$. Find its speed at time $t = 1$.

Solution: $x'(t) = 3t^2 - 1$ and $y'(t) = 4t^3$. (**4 points**) The speed at time $t = 1$ is $\sqrt{x'(1)^2 + y'(1)^2} = \sqrt{2^2 + 4^2} = \boxed{\sqrt{20}}$ (**4 points**)

MATH 180 Quiz 3 (Version II) Solution **Radford**

10/22/04

1. (12 points) Let $y = f(x)$ be defined implicitly by the equation $2x^3y + y^2 + x = 2$.

   a) Find $\dfrac{dy}{dx}$ or y'.

   Solution: By implicit differentiation $6x^2y + 2x^3y' + 2yy' + 1 = 0$. Therefore $(2x^3 + 2y)y' = -(1 + 6x^2y)$ and $\boxed{\dfrac{dy}{dx} = y' = -\dfrac{1 + 6x^2y}{2x^3 + 2y}.}$ (**6 points**)

   b) Find an equation of the line tangent to the graph of $y = f(x)$ at the point $(-1, 3)$.

   Solution: By part a) $y'(-1) = -\dfrac{1 + 6(-1)^2 3}{2(-1)^3 + 2(3)} = -\dfrac{19}{4}$. Therefore

   $\boxed{y - 3 = -\dfrac{19}{4}(x + 1) \text{ or } y = -\dfrac{19}{4}x - \dfrac{7}{4}}$ (**6 points**)

2. (8 points) A particle is moving along a curve in the plane which is parameterized by $x = t^3 + 2t$ and $y = t^4 - t + 2$. Find its speed at time $t = 1$.

Solution: $x'(t) = 3t^2 + 2$ and $y'(t) = 4t^3 - 1$. (**4 points**) The speed at time $t = 1$ is $\sqrt{x'(1)^2 + y'(1)^2} = \sqrt{3^2 + 5^2} = \boxed{\sqrt{34}}$ (**4 points**)