Math 215 **Written Homework 8 Solution** **08/08/08**

1. (**25 points total**)

(a) Suppose n is a perfect square. Then $n = a^2$ for some $a \in \mathbf{Z}$. Write $a = 5\ell + r$, where $\ell, r \in \mathbf{Z}$ and $0 \leq r < 5$. (**3 points**) Now

$$n = a^2 = (5\ell + r)^2 = 5^2\ell^2 + 2{\cdot}5\ell r + r^2 = 5(5\ell^2 + 2\ell r) + r^2. \text{ (\textbf{3 points})}$$

If $r = 0$, 1, or 2 then $n = 5m$, $5m + 1$, or $5m + 4$ respectively with $m = 5\ell^2 + 2\ell r$. (**3 points**) If $r = 3$ then $r^2 = 9 = 5{\cdot}1 + 4$ so $n = 5m + 4$, where $m = 5\ell^2 + 2\ell r + 1$. (**3 points**) If $r = 4$ then $r^2 = 16 = 5{\cdot}3 + 1$ so $n = 5m + 1$, where $m = 5\ell^2 + 2\ell r + 3$. (**3 points**)

(b) $288 = 5{\cdot}57 + 3$; thus 288 is not a perfect square by part (a). (**5 points**)

(c) $2369 = 5{\cdot}473 + 4$; thus the test for perfect square by part (a) is inconclusive. However $2369 = 3{\cdot}789 + 2$; thus 2369 is not a perfect square by Proposition 15.2.3. (**5 points**)

2. (**25 points total**) Suppose that $n \in \mathbf{Z}$ and 7 divides n^2. Write $n = 7m+r$, where $m, r \in \mathbf{Z}$ and $0 \leq r < 7$. Then $n^2 = (7m + r)^2 = 7^2m^2 + 2{\cdot}7mr + r^2 = 7(7m^2 + 2mr) + r^2$. Since 7 divides n^2 necessarily 7 divides r^2. (**15 points**) Using the table

r	0	1	2	3	4	5	6
r^2	0	1	4	9	16	25	36

we see that 7 divides r^2 only when $r = 0$. Therefore $r = 0$ and $n = 7m$. (**10 points**)

3. (**25 points total**) (a) $293 = 27{\cdot}10 + 23$; $q = 10$ and $r = 23$ (**10 points**) (b) $-2931 = 17{\cdot}(-173) + 10$; $q = -173$ and $r = 10$. (**15 points**)

4. (**25 points total**)

(a)

$$\begin{aligned} \mathbf{89} &= \mathbf{17}\cdot 5 + \mathbf{4} \\ \mathbf{17} &= \mathbf{4}\cdot 4 + \mathbf{1} \\ \mathbf{4} &= \mathbf{1}\cdot 4 + 0 \end{aligned}$$

Therefore the greatest common divisor of 89 and 17 is 1. (**15 points**)

(b)

$$\begin{aligned} \mathbf{298} &= \mathbf{8}\cdot 37 + \mathbf{2} \\ \mathbf{8} &= \mathbf{2}\cdot 4 + 0 \end{aligned}$$

Therefore the greatest common divisor of 298 and 8 is 2. (**10 points**)