1. **20 points total** We construct separate truth tables (adding extra columns for convenience):

(a) (**4 points**)

| P | Q | not P | not Q | (not P) or (not Q) |
|---|---|---|---|---|
| T | T | F | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

(b) (**4 points**)

| P | Q | P or Q | not (P or Q) |
|---|---|---|---|
| T | T | T | F |
| T | F | T | F |
| F | T | T | F |
| F | F | F | T |

(c) (**4 points**)

| P | Q | P and Q | not (P and Q) |
|---|---|---|---|
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

(d) (**4 points**)

| P | Q | not P | (not P) and Q |
|---|---|---|---|
| T | T | F | F |
| T | F | F | F |
| F | T | T | T |
| F | F | T | F |

The only locally equivalent statements are those of parts (a) and (c). (**4 points**)

2. **20 points total** We combine truth tables for convenience:

| P | Q | not P | not Q | P or (not Q) | (not P) and Q |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | F | T | T | F |
| F | T | T | F | F | T |
| F | F | T | T | T | F |

(**8 points**)

(a) Since columns for "P or (not) Q" and "(not P) or Q" are not identical, these statements are not equivalent. (**4 points**)

(b) Since column for "P or (not) Q" is the column for "(not P) and Q" with T instead of F and vice versa, these statements are negations of each other. (**4 points**)

(c) Row 1, 2, or 4 of the table shows that "P or (not) Q" does not imply "(not P) and Q" and row 3 of the same shows that "(not P) and Q" does not imply "P or (not) Q".(**4 points**)

3. **20 points total** To show that "S implies T" is false we need only find an instance in which T is false and S is true. Let S be "(P implies Q) implies R" and T be "P implies (Q implies R)".

If P, R are false and Q is true then S is false and T is true. Thus T does not imply S. If P is true and Q, R are false, then S is true and T is false. Therefore S does not imply T.

We have done part (b). (**10 points**) Since one of "S implies T" and "T implies S" is false (both are) then "S is equivalent to T" is false. (**10 points**)

*Comment*: We have shown that "implies" is not an associative operation. Since an implication is not equivalent to its converse "implies" is also not a commutative operation. This exercise is somewhat theoretical. A cardinal point: "P implies Q" is false exactly when P is true and Q is false.

4. **20 points total** Let $P(x)$ be "$x \geq 0$" and $Q(x)$ be "$x^2 > x$", or equivalently $x(x-1) > 0$.

(a)

| | P(x) | Q(x) |
|---|---|---|
| $x < 0$ | F | T |
| $x = 0$ | T | F |
| $0 < x < 1$ | T | F |
| $x = 1$ | T | F |
| $1 < x$ | T | T |

(**8 points**)

A simpler table is obtained by combining lines 2–4 into the case $0 \leq x \leq 1$

Since there is at least one instance in which $P(x)$ is true and $Q(x)$ is false (for example $x = 0$) the universal statement is false. (**8 points**)

(b) The converse of the universal statement is Q(x) implies P(x) for all real numbers $x$. This is false since for $x = -1$ the statement Q(x) is true for but P(x) is false. A truth table for the universal statement:

| | Q(x) | P(x) |
|---|---|---|
| $x < 0$ | T | F |
| $x = 0$ | F | T |
| $0 < x < 1$ | F | T |
| $x = 1$ | F | T |
| $1 < x$ | F | F |

(**4 points**)

5. **20 points total** $P$ is the statement "$a > 4$" and Q is the statement "$a^2 - 3a - 4 \geq 0$". Note that $a^2 - 3a - 4 = (a+1)(a-4)$.

(a) If $a > 4$ then $a^2 - 3a - 4$ is the product of two positive numbers hence is positive. Thus "P implies Q" is true. (**4 points**)

(b) “Q only if P” is equivalent to “Q implies P”. This is false. For example take $a = -1$. Then Q is true as $a^2 - 3a - 4 = 0$ but P is false since $a \not> 4$. (**4 points**)

(c) “P is necessary for Q” is equivalent to “Q implies P”. False by part (b). (**4 points**)

(d) “P if and only if Q” is equivalent to “(P implies Q) and (Q implies P)”. Since “Q implies P” is false by part (b), “P if and only if Q” is false. (**4 points**)

(e) “P is sufficient for Q” is equivalent to “P implies Q”. Thus “P is sufficient for Q” is true by part (a). (**4 points**).