

## Written Homework # 3

Due at the beginning of class 07/07/08

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1. Prove that  $\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$  for all  $n \geq 1$  by induction.
  2. We establish that  $n^2 < 2^n$  for large enough  $n$ .
    - (a) Prove by induction that  $n + 1 \leq 2^{n-1}$  for  $n \geq 3$ .
    - (b) Use part (a) to prove that  $n^2 < 2^n$  for all  $n > 4$ .
    - (c) Find all  $n$  such that  $1 \leq n$  and  $n^2 < 2^n$  is false.
  3. Let  $A, B$  be sets. Working from definitions, show that  $A \cup B \subseteq A$  only if  $B \subseteq A$ .
  4. Let  $A, B$  be sets. Working from definitions, show that  $B \subseteq A \cap B$  if and only if  $B \subseteq A$ .