Math 215 **Summer 2008** **Radford**

Written Homework # 6

Due at the beginning of class 07/25/08

Let A be a subset of the real numbers $\mathbf{R}$. A *maximum for* A is a number $a \in A$ such that $a' \leq a$ for all $a' \in A$. A *minimum for* A is a number $a \in A$ such that $a \leq a'$ for all $a' \in A$. We set $-A = \{-a \,|\, a \in A\}$.

1. Suppose that A is a subset of $\mathbf{R}$.

   (a) Show that A has at most one maximum.

   (b) Let $a \in A$. Show that a is a minimum for A if and only if $-a$ is a maximum for $-A$.

   (c) Use parts (a) and (b) to show that A has at most one minimum.

2. Suppose that A, B are subsets of $\mathbf{R}$.

   (a) Suppose that $A \cup B$ has a maximum. Show that either A has a maximum or B has a maximum.

   (b) Suppose that A and B each has a maximum. Show that $A \cup B$ has a maximum.

3. Let U be a universal set and $A, B \subseteq U$.

   (a) Show that $\chi_{A \cap B} = \chi_A \chi_B$.

   (b) Show that $\chi_{A^c} = 1 - \chi_A$.

   (c) Use parts (a) and (b) to show that $\chi_{A-B} = \chi_A(1 - \chi_B)$.

4. We continue with Problem 3.

(a) Show that $\chi_{A\cup B} = \chi_A + \chi_B - \chi_{A\cap B}$ ($= \chi_A + \chi_B - \chi_A\chi_B$ by part (a) of Problem 3).

(b) Using the fact that $A = B$ if and only if $\chi_A = \chi_B$, use parts (a) and (b) of Problem 3 and part (a) to prove De Morgan's Law $(A\cup B)^c = A^c \cap B^c$.

5. Find the greatest common divisor of

(a) 22 and 234;

(b) 39 and 385;

(c) 16 and 120.