Math 215, Fall 05 Homework #1 Solution

09/09/05

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1. (**20 points total**) We add extra columns to the truth table in some cases for convenience.

a)

P	Q	not P	not Q	(not P) and (not Q)
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

b)

P	Q	P and Q	not (P and Q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

c)

P	Q	not P	not Q	(not P) or (not Q)
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

d)

P	Q	P or Q	not (P or Q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Thus the statements of a) and d) are logically equivalent and the statements of b) and c) are logically equivalent. (**Tables: 10 points; Conclusions: 10 points.**)

2. (**20 points total**) We add extra columns to the truth table for convenience.

a)

P	Q	not P	(not P) and Q
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

b)

P	Q	not Q	P or (not Q)
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

a) These statements are not logically equivalent, b) they are negations of the other, and c) neither implies each other. (**Tables: 5 points; Conclusions: 15 points.**)

3. (**20 points total**) We add extra columns to the truth table for convenience.

a)

P	Q	not P	(not P) implies Q
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

b)

P	Q	P implies Q	not (P implies Q)
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

c)

P	Q	not Q	(not Q) implies P
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

Thus the statements of a) and c) are equivalent, and the statement of b) implies the statements of a), c). To see the latter note that the statement of b) true implies that the statement of a) true, and not vice versa. This is the essential observation in the construction of the truth table of

(not (P implies Q)) implies ((not P) implies Q).

Also note that the statements of a) and c) are contrapositives and thus equivalent. (**Tables: 10 points; Conclusions: 10 points.**)

4. We label statements P : $x \geq 0$ and Q : $x^2 < x$.

a)

	P	Q	P implies Q
$x < 0$	F	F	T
$x = 0$	T	F	F
$0 < x < 1$	T	T	T
$1 \leq x$	T	F	F

Thus $x \geq 0$ does not imply $x^2 < x$.

b) The converse of the statement of part a), P implies Q, is Q implies P. We can use the table of part a) to construct

	Q	P	Q implies P
$x < 0$	F	F	T
$x = 0$	F	T	T
$0 < x < 1$	T	T	T
$1 \leq x$	F	T	T

Thus $x^2 < x$ implies $x \geq 0$. There is a simpler table, namely

	Q	P	Q implies P
$x < 0$	F	F	T
$x \geq 0$		T	T

Comment: For this problem basic arithmetic is granted to work out the necessary calculations for the various cases.

5. (**20 points total**) P OR Q is logically equivalent to ((not P) and Q) or (P and (not Q)) as the following truth table shows.

P	Q	not P	not Q	(not P) and Q	P and (not Q)	((not P) and Q) or (P and (not Q))
T	T	F	F	F	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	F	F	F	F

A popular answer was (P or Q) and (not(P and Q)). One can deduce that the two are equivalent by noting that "and" distributes over "or"; that is

P and (Q or R) is logically equivalent to (P and Q) or (P and R).