

# Math 215, Fall 05

# Homework #2

Due Friday, 09/09/05 at the beginning of class

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Some of the exercises below depend on axioms for the real number system which we take to be:

- Axiom 1 *Commutative Laws*:  $a + b = b + a$  and  $ab = ba$  for all  $a, b \in \mathbf{R}$ ;
- Axiom 2 *Associative Laws*:  $(a + b) + c = a + (b + c)$  and  $(ab)c = a(bc)$  for all  $a, b, c \in \mathbf{R}$ ;
- Axiom 3 *Distributive Law(s)*:  $a(b + c) = ab + ac$  and  $(a + b)c = ac + bc$  for all  $a, b, c \in \mathbf{R}$ ;
- Axiom 4 *Existence of a Neutral Element for Addition*: There is an element  $0 \in \mathbf{R}$  such that  $a + 0 = a = 0 + a$  for all  $a \in \mathbf{R}$ ;
- Axiom 5 *Existence of a Neutral Element for Multiplication*: There is an element  $1 \in \mathbf{R}$  such that  $a1 = a = 1a$  for all  $a \in \mathbf{R}$ ;
- Axiom 6 *Existence of a Additive Inverses*: For all  $a \in \mathbf{R}$  there is an  $a' \in \mathbf{R}$  such that  $a + a' = 0 = a' + a$ .
- Axiom 7 *Existence of a Multiplicative Inverses*: For all nonzero  $a \in \mathbf{R}$  there is an  $a'' \in \mathbf{R}$  such that  $aa'' = 1 = a''a$ .

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1. Only using Axioms 1–7 prove:

**Proposition 1** *Let  $\mathbf{R}$  be the real number system. Then:*

- a) For all  $a \in \mathbf{R}$  there exists a unique  $a' \in \mathbf{R}$  such that  $a + a' = 0 = a' + a$ .
- b) For all  $a, b \in \mathbf{R}$  there is a unique solution  $x \in \mathbf{R}$  to  $a + x = b$ .
- c)  $a0 = 0a = 0$  for all  $a \in \mathbf{R}$ .

The element  $a'$  of part a) is denoted  $-a$ .

2. Only using Axiom 1–7 and Proposition 1 prove that  $a(-b) = -(ab) = (-a)b$  for all  $a, b \in \mathbf{R}$ .

3. Only using Axioms 1–7 show that there is *at most* one element  $e \in \mathbf{R}$  such that  $ae = a = ea$  for all  $a \in \mathbf{R}$ .

4. Let  $a \in \mathbf{R}$ , let  $P$  denote the statement “ $a > 3$ ”, let and  $Q$  denote the statement “ $a^2 - 2a - 3 > 0$ ”. Which of the following are true? In each case supply a proof or counterexample. For proofs you may assume Axiom 3.1.2 on page 24 of the text.

- a)  $P$  implies  $Q$ .
- b)  $Q$  only if  $P$ .
- c)  $P$  if and only if  $Q$ .
- d)  $P$  is necessary for  $Q$ .
- e)  $P$  is sufficient for  $Q$ .