Math 215, Fall 05 Homework #5 (REVISION)

Due Friday, 10/07/05 at the beginning of class. **Problem 3.b) clarified.**

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1. We define the union of sets $A_1, \ldots, A_n$ inductively by

$$A_1 \cup \cdots \cup A_n = \begin{cases} A_1 & : \quad n = 1; \\ (A_1 \cup \cdots \cup A_{n-1}) \cup A_n & : \quad n > 1. \end{cases}$$

a) Suppose that A, B and C are sets and $A, B \subseteq C$. Use the definition of subset to prove that $A \cup B \subseteq C$.

b) Suppose that $A_1, \ldots, A_n$ are sets and $A_1, \ldots, A_n \subseteq C$. Construct a proof by induction to show that $A_1 \cup \cdots \cup A_n \subseteq C$.

2. Describe the power set $P(S)$ in the following cases by listing its elements: a) $S = \emptyset$, b) $S = \{1\}$, c) $S = \{\emptyset\}$, d) $S = \{8, 5\}$, and e) $S = P(\{1\})$.

3. Let U denote a universal set and let $A, B \subseteq U$.

a) Show that $(A^c)^c = A$ by completing the following truth table

$x \in A$	$x \in A^c$	$x \in (A^c)^c$

to show that $x \in A$ if and only if $x \in (A^c)^c$.

b) Using only part a), show that De Morgan's Law $(A \cup B)^c = A^c \cap B^c$ is logically equivalent to his law $(A \cap B)^c = A^c \cup B^c$. [Hint: $(A \cup B)^c = ((A^c)^c \cup (B^c)^c)^c = \cdots$.]

4. Let U denote a universal set and let $A, B \subseteq U$. Establish De Morgan's Law $(A \cup B)^c = A^c \cap B^c$ by completing the truth table

$x \in A$	$x \in B$	$x \in A \cup B$	$x \in (A \cup B)^c$	$x \in A^c$	$x \in B^c$	$x \in A^c \cap B^c$

to show that $x \in (A \cup B)^c$ if and only if $x \in A^c \cap B^c$.