

Math 215, Fall 05

Homework #8

Due Friday, 10/28/05 at the beginning of class

Let U be a set, suppose $f, g : U \rightarrow \mathbf{R}$ are functions from U to the set of real numbers \mathbf{R} , and let $a \in \mathbf{R}$. Then $f + g, fg, af : U \rightarrow \mathbf{R}$ are defined by

$$(f + g)(x) = f(x) + g(x),$$

$$(fg)(x) = f(x)g(x),$$

and

$$(af)(x) = a(f(x))$$

for all $x \in \mathbf{R}$. If $a \in \mathbf{R}$, by abuse of notation we regard a as the constant function from U to \mathbf{R} defined by $a(x) = a$ for all $x \in U$.

Now regard U as a universal set. For a subset A of U the *characteristic function of A* is the function $\chi_A : U \rightarrow \mathbf{R}$ defined by

$$\chi_A(x) = \begin{cases} 1 & : x \in A; \\ 0 & : x \notin A. \end{cases}$$

1. Let U be a universal set.

- a) Let $A, B \subseteq U$. Show that $\chi_A = \chi_B$ if and only if $A = B$.
- b) Show that $\chi_\emptyset = 0$ and $\chi_U = 1$
- c) Suppose that $f : U \rightarrow \mathbf{R}$ is a function such that $\text{Im } f \subseteq \{0, 1\}$. Show that f is the characteristic function of some subset of U .

2. Let U be a universal set and $A, B \subseteq U$.

- a) Show that $\chi_{A^c} = 1 - \chi_A$.
- b) Show that $\chi_{A \cap B} = \chi_A \chi_B$.

- c) Express $\chi_{A \cup B}$ and χ_{A-B} in terms of χ_A and χ_B . Justify your formulas.
3. Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be functions.
- a) Suppose that f and g are injective. Show that $g \circ f$ is injective.
 - b) Suppose that f and g are surjective. Show that $g \circ f$ is surjective.
 - c) Suppose that f and g are bijective. Then $g \circ f$ is bijective by parts a) and b). Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
4. Let $f : \mathbf{R} \longrightarrow \mathbf{R}$ be the function defined by $f(x) = x^2 + 3x - 4$. for all $x \in \mathbf{R}$.
- a) Determine whether or not f is injective.
 - b) Determine whether or not f is surjective.
 - c) Find $f^{-1}(\{0, 75/4\})$.
 - d) Find $f([0, 1])$. [Hint: You may assume from Calculus that the image of an interval under f (which is continuous) is an interval.]