

Math 215, Fall 05 Homework #10

Solution

11/11/05

1. (**20 points total**) This exercise is a direct application of binomial symbols interpreted as number of subsets.

a) There are $\binom{8}{5} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$ such committees. (**5 points**)

b) Since two particular individuals are to be included, the committee is formed by choosing the remaining 3 from $8 - 2 = 6$, and these three determine the committee. Thus there are $\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$ such committees. (**5 points**)

c) Since two particular individuals are to be excluded, the committee is formed by choosing the 5 from a pool of $8 - 2 = 6$ individuals. Thus there are $\binom{6}{5} = 6$ such committees. (**5 points**)

d) The committees where this is *not* the case are the committees where both are excluded. Since the total number of 5 person committees is 56 by part a), and the number of committees where two particular individuals are excluded is 6 by part c), the answer is $56 - 6 = 50$. (**5 points**)

[Note: This problem could be done by the inclusion-exclusion principle as well. Let X be the set of committees which includes the first person and let Y be the set of committees which include the second. Then $X \cup Y$ is the set of committees which include one or the other and $X \cap Y$ is the set of committees which include both. The number we wish to calculate is

$$|X \cup Y| = |X| + |Y| - |X \cap Y| = \binom{7}{4} + \binom{7}{4} - \binom{6}{3} = 35 + 35 - 20 = 50.$$

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2. (**20 points total**) This is a direct application of the binomial theorem.

a) $\binom{14}{10} = \frac{14 \cdot 13 \cdot 12 \cdot 11}{4 \cdot 3 \cdot 2 \cdot 1} = 7 \cdot 13 \cdot 11 = 1001$ (**10 points**)

b) $\binom{14}{8} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7 \cdot 13 \cdot 11 \cdot 3 = 3003$ (**10 points**)

3. (**20 points total**) Here a systematic listing of certain types of functions is required.

a) $|\text{Fun}(A, B)| = |B|^{|A|} = 2^3 = 8$. (**5 points**)

b) The only non-surjections are the constant functions. Therefore there are $8 - 2 = 6$ surjections. These are given by the following amalgamation of tables:

image	1	2	
f_1 inputs	1	2, 3	. (5 points)
f_2 inputs	2	1, 3	
f_3 inputs	3	1, 2	
f_4 inputs	2, 3	1	
f_5 inputs	1, 3	2	
f_6 inputs	1, 2	3	

c) $|\text{Fun}(B, A)| = |A|^{|B|} = 3^2 = 9$. (**5 points**)

d) There are $3 \cdot 2 = 6$ injections. These are given by the following amalgamation of tables:

image	1	2	3	
f_1 inputs	9	11		. (5 points)
f_2 inputs	9		11	
f_3 inputs		9	11	
f_4 inputs	11	9		
f_5 inputs	11		9	
f_6 input		11	9	

4. (**20 points total**) This problem involves several counting principles; counting ordered pairs, counting subsets, and using the two in combination.

a) Each of the four makes an independent choice of one out of seven: $7^4 = 2401$. **(6 points)**

b) Let us consider exhaustive and mutually exclusive cases.

Case 1: 4 choices made. The choices can be considered 4-element subsets of a 7-element set. Thus there are $\binom{7}{4} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$ possibilities.

Case 2: 3 choices made. Then 2 people make the same choice and the 2 others make different choices from the 6 items not chosen by the first 2. The combination is $7 \cdot \binom{6}{2} = 7 \cdot \frac{6 \cdot 5}{2 \cdot 1} = 105$.

Case 3: 2 choices made. Here 2 make the same choice and the other 2 make the same choice from the 6 items not chosen by the first 2, or 3 make the same choice and the fourth makes a different choice. The number of possibilities in the first instance is $\binom{7}{2} = \frac{7 \cdot 6}{2 \cdot 1} = 21$ and in the second instance $7 \cdot 6 = 42$.

Case 4: 1 choice made. In this case there are 7 possibilities.

The total number of choices is therefore

$$35 + 105 + (21 + 42) + 7 = 210. \quad \textbf{(14 points)}$$