Math 215, Fall 05 Homework #11

Due Wednesday, 11/23/05 at the beginning of class.

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1. Let p be a positive prime integer. Show that there is no rational number r such that $p = r^2$. [You may use the fact that if $a, b \in \mathbf{Z}$ then $p|ab$ implies $p|a$ or $p|b$, and also the fact that any non-zero rational number r can be written $r = a/b$, where $a, b \in \mathbf{Z}$ and the only positive common divisor of a and b is 1.]

2. Write the following as rational numbers:

   a) $0.7\overline{31}$;

   b) $12.\overline{8697}$.

3. Prove Corollary 14.2.4 from Proposition 14.2.3 by induction. Use the definition of Problem 2, Homework # 7 to define A^n ($A_1 = \cdots = A_n = A$ in this case).

4. Let $f : \mathbf{Z}^+ \longrightarrow \mathbf{Z}$ be the function defined by

$$f(n) = \begin{cases} \dfrac{n}{2} & : \ n \text{ even;} \\[2ex] -\left(\dfrac{n-1}{2}\right) & : \ n \text{ odd.} \end{cases}$$

The following table indicates the enumeration scheme behind the definition of the function.

n	$\cdots$	7	5	3	1	2	4	6	8	$\cdots$
$f(n)$	$\cdots$	-3	-2	-1	0	1	2	3	4	$\cdots$

Show that f is a bijection. [You may assume basic facts about the integers, in particular any integer n can be written $n = 2m$ for some integer m or $n = 2m + 1$ for some integer m, but not both, and in either case the integer m is unique.]