Math 215, Fall 05 Homework #12

Due Friday, 12/02/05 at the beginning of class.

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1. Let d, a, b, r, and q be integers.

   a) Suppose that $d|a$ and $d|b$. Show that $d|(ra + qb)$.

   b) Suppose $a = qb + r$. Show that the set of common divisors of a and b is the set of common divisors of b and r.

2. Let a, b be positive integers, and write $a = qb + r$, where $q, r \in \mathbf{Z}$ and $0 \leq r < b$. Suppose that $d = \gcd(a, b)$.

   a) If $r = 0$ show that $d = b$.

   b) If $r > 0$ show that $d = \gcd(b, r)$.

4. Use Problem 2 to find:

   a) $\gcd(100, 3)$;

   b) $\gcd(100, 82)$.

4. Let p and a be positive integers and suppose that $p|a^2$.

   a) Show that $p|(ra + sp)^2$ for all integers r, s.

   b) Use part a), the definition of prime integer, and Theorem 15.1.1 to construct a proof by induction that $p|a$. [Hint: If $a \leq p$ consider $p = qa + r$, where $0 \leq r < a$. If $p < a$ consider $a = qp + r$, where $0 \leq r < p$.]