MATH 220 # Quiz 1 (version 1) **Radford 09/12/03**

Name (print) _____

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (15 pts.) A brine solution is flowing into a tank containing 60 gallons at the rate of 5 gallons per minute and is flowing out at the same rate. Suppose that the solution flowing in contains 30% salt. Given that the tank is kept well stirred and contains pure water initially, find the amount of salt x in the tank at time t.

Solution: Let x be the amount of salt in the tank at time t. Then

$$\frac{dx}{dt} = (5)(.30) - 5\left(\frac{x}{60}\right) = 1.5 - \frac{x}{12},$$

or

$$\frac{dx}{dt} + \frac{x}{12} = 1.5, \qquad \text{and} \qquad x(0) = 0 \ \ (5 \text{ points}).$$

Therefore $\mu = e^{\int \frac{1}{12}\,dx} = e^{\frac{x}{12}}$ and

$$x = \frac{1}{\mu}\left(\int \mu(1.5)\,dx + C\right) = \frac{1}{e^{\frac{x}{12}}}\left(12e^{\frac{x}{12}}(1.5)\,dx + C\right) = 18 + Ce^{-\frac{x}{12}} \ \ (5 \text{ points}).$$

Since $x(0) = 0$ it follows that $C = -18$ and therefore

$$x = 18(1 - e^{-\frac{x}{12}}) \ \ (5 \text{ points}).$$

2. (5 pts.) An object of mass 30 kg is released from 500 meters above the ground encounters a constant air resistance of $5-\text{N sec/m}$. Let v be the velocity of the object. Express $\dfrac{dv}{dt}$ as a function of v.

Solution: The equation to work from is

$$m\frac{dv}{dt} = mg - bv,$$

or

$$\frac{dv}{dt} = g - \frac{b}{m}v,$$

where in this case $m = 30$, $g = 9.81$ and $b = 5$. Therefore

$$\frac{dv}{dt} = 9.81 - \frac{v}{6} \ \ (5 \text{ points}).$$

MATH 220 # Quiz 1 (version 2) **Radford 09/12/03**

Name (print) _____

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (15 pts.) A brine solution is flowing into a tank containing 40 gallons at the rate of 7 gallons per minute and is flowing out at the same rate. Suppose that the solution flowing in contains 20% salt. Given that the tank is kept well stirred and contains pure water initially, find the amount of salt x in the tank at time t.

Solution: Let x be the amount of salt in the tank at time t. Then

$$\frac{dx}{dt} = (7)(.20) - 7\left(\frac{x}{40}\right) = 1.4 - \left(\frac{7}{40}\right)x,$$

or

$$\frac{dx}{dt} + \left(\frac{7}{40}\right)x = 1.4, \qquad \text{and} \qquad x(0) = 0 \ \ (5 \text{ points}).$$

Therefore $\mu = e^{\int \frac{7}{40}\,dx} = e^{\frac{7x}{40}}$ and

$$x = \frac{1}{\mu}\left(\int \mu(1.4)\,dx + C\right) = \frac{1}{e^{\frac{7x}{40}}}\left(\frac{40}{7}e^{\frac{7x}{40}}(1.4)\,dx + C\right) = 8 + Ce^{-\frac{7x}{40}} \ \ (5 \text{ points}).$$

Since $x(0) = 0$ it follows that $C = -8$ and therefore

$$x = 8(1 - e^{-\frac{7x}{40}}) \ \ (5 \text{ points}).$$

2. (5 pts.) An object of mass 50 kg is released from 700 meters above the ground encounters a constant air resistance of 8−N sec/m. Let v be the velocity of the object. Express $\dfrac{dv}{dt}$ as a function of v.

Solution: The equation to work from is

$$m\frac{dv}{dt} = mg - bv,$$

or

$$\frac{dv}{dt} = g - \frac{b}{m}v,$$

where in this case $m = 50$, $g = 9.81$ and $b = 8$. Therefore

$$\frac{dv}{dt} = 9.81 - \frac{4}{25}v \ \ (5 \text{ points}).$$