MATH 220 # Quiz 2 (version 1) **Radford 09/26/03**

Name (print) ______________________________

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (12 pts.) Find the unique solution to $y'' - y' - 6y = 0$, where $y(0) = 2$ and $y'(0) = -3$.

Solution: The auxiliary equation is $r^2 - r - 6 = 0$. Since $r^2 - r - 6 = (r-3)(r+2)$ the auxiliary equation has two distinct real roots $r = 3, -2$ (3 points). Therefore the general solution is

$$y = c_1 e^{3x} + c_2 e^{-2x}. \text{(4 points)}$$

Since

$$y' = 3c_1 e^{3x} - 2c_2 e^{-2x}$$

we need to solve the system

$$\begin{aligned} 2 &= y(0) = c_1 + c_2 \\ -3 &= y'(0) = 3c_1 - 2c_2 \end{aligned}$$

which has solution

$$c_1 = \frac{1}{5} \quad \text{and} \quad c_2 = \frac{9}{5}.$$

Therefore

$$y = \frac{1}{5}e^{3x} + \frac{1}{5}e^{-2x} \quad \text{(4 points)}.$$

2. (8 pts.) Find the general solution to $y'' + y' + 5y = 0$.

Solution: The auxiliary equation is $r^2+r+5 = 0$ which has complex roots $r = -\frac{1}{2} \pm \frac{\sqrt{19}}{2}\imath$ (4 points). Therefore the general solution is

$$y = c_1 e^{-\frac{1}{2}x} \cos(\frac{\sqrt{19}}{2}x) + c_2 e^{-\frac{1}{2}x} \sin(\frac{\sqrt{19}}{2}x). \quad \text{(4 points)}.$$

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1. (12 pts.) Find the unique solution to $y'' - y' - 20y = 0$, where $y(0) = -1$ and $y'(0) = 2$.

Solution: The auxiliary equation is $r^2 - r - 20 = 0$. Since $r^2 - r - 20 = (r-5)(r+4)$ the auxiliary equation has two distinct real roots $r = 5, -4$ (4 points). Therefore the general solution is

$$y = c_1 e^{5x} + c_2 e^{-4x}. (4 \text{ points})$$

Since

$$y' = 5c_1 e^{3x} - 4c_2 e^{-2x}$$

we need to solve the system

$$\begin{aligned} -1 &= y(0) = c_1 + c_2 \\ 2 &= y'(0) = 5c_1 - 4c_2 \end{aligned}$$

which has solution

$$c_1 = -\frac{2}{9} \quad \text{and} \quad c_2 = -\frac{7}{9}.$$

Therefore

$$y = -\frac{2}{9}e^{5x} - \frac{7}{9}e^{-4x} \ (4 \text{ points}).$$

2. (8 pts.) Find the general solution to $y'' + 3y' + 4y = 0$.

Solution: The auxiliary equation is $r^2+3r+4 = 0$ which has complex roots $r = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}\imath$ (4 points). Therefore the general solution is

$$y = c_1 e^{-\frac{3}{2}x} \cos(\frac{\sqrt{7}}{2}x) + c_2 e^{-\frac{1}{2}x} \sin(\frac{\sqrt{7}}{2}x). \ (4 \text{ points}).$$