MATH 220 Quiz 3 (Version 1) **Radford 10/03/03**

Name (print) ________________________________

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (20 pts.) Find the general solution to $y'' + 6y' = 4x + 2$ $(*)$ by:

a) Finding a fundamental set of solutions to $y'' + 6y' = 0$; (5 points)

Solution: The auxiliary equation $r^2 + 6r = r(r + 6) = 0$ has roots $r = 0, -6$. Thus a fundamental set of solutions is $\{1, e^{-6x}\}$.

b) Finding the general solution to the homogeneous equation $y'' + 6y' = 0$; (3 points)

Solution: $y = y_h = c_1 1 + c_2 e^{-6x} = c_1 + c_2 e^{-6x}$.

c) Finding a particular solution to $(*)$ (7 points).

Solution: $y_p = x^s(Ax + B) = x(Ax + B)$ since B is a solution to the homogeneous system and none of the terms of $Ax^2 + Bx$ are. Since $y_p' = 2Ax + B$ and $y_p'' = 2A$ we need to solve

$$4x + 2 = y_p'' + 6y_p' = 2A + 6(2Ax + B) = 12Ax + (2A + 6B)$$

or the linear system

$$\begin{aligned} 12A &= 4 \\ 2A + 6B &= 2 \end{aligned}$$

Therefore

$$A = \frac{1}{3} \qquad \text{and} \qquad B = \frac{2}{9}$$

and

$$y_p = \frac{1}{3}x^2 + \frac{2}{9}x.$$

d) General solution to $(*)$: (5 points).

Solution: $y = y_h + y_p = c_1 + c_2 e^{-6x} + \dfrac{1}{3}x^2 + \dfrac{2}{9}x$.

MATH 220 # Quiz 3 (Version 2) **Radford** **10/03/03**

Name (print) ________________________________

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

1. (20 pts.) Find the general solution to $y'' + 8y' = 3x + 1$ $(*)$ by:

a) Finding a fundamental set of solutions to $y'' + 8y' = 0$; (5 points)

Solution: The auxiliary equation $r^2 + 8r = r(r+8) = 0$ has roots $r = 0, -8$. Thus a fundamental set of solutions is $\{1, e^{-8x}\}$.

b) Finding the general solution to the homogeneous equation $y'' + 8y' = 0$; (3 points)

Solution: $y = y_h = c_1 1 + c_2 e^{-8x} = c_1 + c_2 e^{-8x}$.

c) Finding a particular solution to $(*)$; (7 points).

Solution: $y_p = x^s(Ax + B) = x(Ax + B)$ since B is a solution to the homogeneous system and none of the terms of $Ax^2 + Bx$ are. Since $y_p' = 2Ax + B$ and $y_p'' = 2A$ we need to solve

$$3x + 1 = y_p'' + 8y_p' = 2A + 8(2Ax + B) = 16Ax + (2A + 8B)$$

or the linear system

$$\begin{aligned} 16A &= 3 \\ 2A + 8B &= 1 \end{aligned}$$

Therefore

$$A = \frac{3}{16} \qquad \text{and} \qquad B = \frac{5}{64}$$

and

$$y_p = \frac{3}{16}x^2 + \frac{5}{64}x.$$

d) General solution to $(*)$: (5 points)

Solution: $y = y_h + y_p = c_1 + c_2 e^{-8x} + \dfrac{3}{16}x^2 + \dfrac{5}{64}x$.