**MATH 220** # Quiz 4 (Version I) **Radford** **10/17/03**

**Name (print)** ______________________________

**Discussion (circle day, time) Tu Th 10 12**

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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Consider the system of equations

$$\begin{aligned} x' + 2x + 7y &= 0 \\ y' - 6y &= 0. \end{aligned}$$

1. (8 pts.) Write the system in the form

$$\begin{aligned} L_1[x] + L_2[y] &= 0 \\ L_3[x] + L_4[y] &= 0, \end{aligned}$$

where $L_1, \ldots, L_4$ are polynomials in $D = \dfrac{d}{dt}$.

*Solution:*

$$\begin{aligned} (D + 2I)[x] + (7I)[y] &= 0 \quad (\mathbf{4\ \ points}) \\ 0[x] + (D - 6I)[y] &= 0. \quad (\mathbf{4\ \ points}) \end{aligned}$$

2. (12 pts.) Use the reformulation of the system described in Problem 1 to eliminate the $y$ term and then solve for $x$.

*Solution:* Generally

$$(L_1 \circ L_4 - L_2 \circ L_3)[x] = 0,$$

so, as $L_3 = 0$,

$$(D + 2I) \circ (D - 6I)[x] = 0.$$

Therefore $(D^2 - 4D - 12I)[x] = 0$ (**6 points**), or

$$x'' - 4x' - 12x = 0.$$

The auxilliary equation is $0 = r^2 - 4r - 12 = (r + 2)(r - 6)$. Therefore the solution for $x$ is

$$x = c_1 e^{-2t} + c_2 e^{6t} \quad (\mathbf{6\ \ points}).$$

**MATH 220** Quiz 4 (Version II) **Radford** **10/17/03**

**Name (print)** ______________________________

**Discussion (circle day, time) Tu Th 10 12**

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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Consider the system of equations

$$\begin{aligned} x' + 3x + 4y &= 0 \\ y' - 5y &= 0. \end{aligned}$$

1. (8 pts.) Write the system in the form

$$\begin{aligned} L_1[x] + L_2[y] &= 0 \\ L_3[x] + L_4[y] &= 0, \end{aligned}$$

where $L_1, \ldots, L_4$ are polynomials in $D = \dfrac{d}{dt}$.

*Solution:*

$$\begin{aligned} (D + 3I)[x] + (4I)[y] &= 0 \quad (\mathbf{4\ \ points}) \\ 0[x] + (D - 5I)[y] &= 0. \quad (\mathbf{4\ \ points}) \end{aligned}$$

2. (12 pts.) Use the reformulation of the system described in Problem 1 to eliminate the $y$ term and then solve for $x$.

*Solution:* Generally

$$(L_1 \circ L_4 - L_2 \circ L_3)[x] = 0,$$

so, as $L_3 = 0$,

$$(D + 3I) \circ (D - 5I)[x] = 0.$$

Therefore $(D^2 - 2D - 15I)[x] = 0$ (**6 points**), or

$$x'' - 2x' - 15x = 0.$$

The auxilliary equation is $0 = r^2 - 2r - 15 = (r + 3)(r - 5)$. Therefore the solution for $x$ is

$$x = c_1 e^{-3t} + c_2 e^{5t} \quad (\mathbf{6\ \ points}).$$