MATH 220 Quiz 5 (Version I) **Radford 11/07/03**

Name (print) ______________________________

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (10 pts.) Determine the Taylor polynomial $p_4(x)$ approximating the solution at $x_0 = 0$ of the initial value problem $y'' = (x+2)^3 + y^2$, where $y(0) = 1$ and $y'(0) = -2$.

Solution: The equation is

$$y^{(2)} = (x+2)^3 + y^2.$$

Therefore

$$\begin{aligned} y^{(3)} &= 3(x+2)^2 + 2yy^{(1)}; \\ y^{(4)} &= 6(x+2) + 2(y^{(1)}y^{(1)} + yy^{(2)}). \end{aligned}$$

Since $y(0) = 1$ and $y^{(1)}(0) = -2$ we have

$$\begin{aligned} y^{(2)}(0) &= 2^3 + (-1)^2 &&= 9; \\ y^{(3)}(0) &= 3(2^2) + 2(1)(-2) &&= 8; \qquad (\mathbf{6} \text{ points}) \\ y^{(4)}(0) &= 6(2) + 2((-2)^2 + (1)(9)) &&= 38. \end{aligned}$$

Therefore

$$p_4(x) = y(0) + y^{(1)}x + \frac{y^{(2)}(0)}{2!}x^2 + \frac{y^{(3)}(0)}{3!}x^3 + \frac{y^{(4)}(0)}{4!}x^4 = 1 - 2x + \frac{9}{2}x^2 + \frac{4}{3}x^3 + \frac{19}{12}x^4. \qquad (\mathbf{4} \text{ points})$$

2. (10 pts.) Suppose that $y = \sum_{n=0}^{\infty} a_n x^n$ is a power series solution to $y' + 5x = 2y$. Express a_1, a_2, and a_3 in terms of a_0.

Solution: Write $y = \sum_{n=0}^{\infty} a_n x^n$. Thus $y' = \sum_{n=0}^{\infty}(n+1)a_{n+1}x^n$. The equation is

$$\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n + 5x = \sum_{n=0}^{\infty} 2a_n x^n. \qquad (\mathbf{3} \text{ points})$$

Comparing coefficients we have

$$\begin{aligned} a_1 &= 2a_0; \\ 2a_2 + 5 &= 2a_1; \\ 3a_3 &= 2a_2. \end{aligned} \qquad \text{Therefore} \qquad \begin{aligned} a_1 &= 2a_0; \\ a_2 &= \frac{1}{2}(2a_1 - 5) &&= 2a_0 - \frac{5}{2}; \qquad (\mathbf{7} \text{ points}) \\ a_3 &= \frac{2}{3}a_2 &&= \frac{4}{3}a_0 - \frac{5}{3}. \end{aligned}$$

MATH 220 Quiz 5 (Version II) **Radford 11/07/03**

Name (print) ______________________________

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (10 pts.) Determine the Taylor polynomial $p_4(x)$ approximating the solution at $x_0 = 0$ of the initial value problem $y'' = (x+1)^3 + y^2$, where $y(0) = 3$ and $y'(0) = -1$.

Solution: The equation is

$$y^{(2)} = (x+1)^3 + y^2.$$

Therefore

$$\begin{aligned} y^{(3)} &= 3(x+1)^2 + 2yy^{(1)}; \\ y^{(4)} &= 6(x+1) + 2(y^{(1)}y^{(1)} + yy^{(2)}). \end{aligned}$$

Since $y(0) = 3$ and $y^{(1)}(0) = -1$ we have

$$\begin{aligned} y^{(2)}(0) &= 1^3 + 3^2 &&= 10; \\ y^{(3)}(0) &= 3(1^2) + 2(3)(-1) &&= -3; \qquad (\mathbf{6} \text{ points}) \\ y^{(4)}(0) &= 6(1) + 2((-1)^2 + 3(10)) &&= 68. \end{aligned}$$

Therefore

$$p_4(x) = y(0) + y^{(1)}x + \frac{y^{(2)}(0)}{2!}x^2 + \frac{y^{(3)}(0)}{3!}x^3 + \frac{y^{(4)}(0)}{4!}x^4 = 3 - x + 5x^2 - \frac{1}{2}x^3 + \frac{17}{6}x^4. \qquad (\mathbf{4} \text{ points})$$

2. (10 pts.) Suppose that $y = \sum_{n=0}^{\infty} a_n x^n$ is a power series solution to $y' + 3x = 4y$. Express a_1, a_2, and a_3 in terms of a_0.

Solution: Write $y = \sum_{n=0}^{\infty} a_n x^n$. Thus $y' = \sum_{n=0}^{\infty} (n+1)a_{n+1}x^n$. The equation is

$$\sum_{n=0}^{\infty} (n+1)a_{n+1}x^n + 3x = \sum_{n=0}^{\infty} 4a_n x^n. \qquad (\mathbf{3} \text{ points})$$

Comparing coefficients we have

$$\begin{aligned} a_1 &= 4a_0; \\ 2a_2 + 3 &= 4a_1; \\ 3a_3 &= 4a_2. \end{aligned} \qquad \text{Therefore} \qquad \begin{aligned} a_1 &= 4a_0; \\ a_2 &= \frac{1}{2}(4a_1 - 3) &&= 8a_0 - \frac{3}{2}; \qquad (\mathbf{7} \text{ points}) \\ a_3 &= \frac{4}{3}a_2 &&= \frac{32}{3}a_0 - 2. \end{aligned}$$