MATH 220 Quiz 8 Version I Solution **Radford** **12/05/03**

Name (print) ____________________________

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

Given that

$$\int x\sin\alpha x\,dx = -\frac{1}{\alpha}(x\cos\alpha x) + \frac{1}{\alpha^2}\sin\alpha x + C$$

and

$$\int x\cos\alpha x\,dx = \frac{1}{\alpha}(x\sin\alpha x) + \frac{1}{\alpha^2}\cos\alpha x + C$$

for $\alpha \neq 0$, find the formal solution to the heat flow problem described by

$$\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2} \text{ for } 0 < x < 4\pi \text{ and } t > 0, \quad u(0,t) = 0 = u(4\pi,t) \text{ for } t > 0, \quad u(x,0) = x \text{ for } 0 < x < 4\pi.$$

Solution: Generally,

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-\beta\left(\frac{n\pi}{L}\right)^2 t} \sin\frac{n\pi}{L}x.$$

Here $\beta = 5$ and $L = 4\pi$. Therefore $\dfrac{n\pi}{L} = \dfrac{n}{4}$ and

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-5\left(\frac{n}{4}\right)^2 t} \sin\frac{n}{4}x. \quad (\textbf{10 points})$$

Using the given we compute

$$b_n = \frac{2}{L}\int_0^{4\pi} x\sin\frac{n}{4}x\,dx = \frac{2}{4\pi}\left[-\frac{1}{\left(\frac{n}{4}\right)}x\cos\frac{n}{4}x + \frac{1}{\left(\frac{n}{4}\right)^2}\sin\frac{n}{4}x\right]_0^{4\pi} = \frac{2}{4\pi}\left(-\frac{4}{n}(4\pi)(\cos n\pi)\right)$$

so

$$b_n = -\frac{8}{n}(-1)^n = \frac{8}{n}(-1)^{n+1}$$

and hence

$$u(x,t) = \sum_{n=1}^{\infty} \frac{8}{n}(-1)^{n+1} e^{-7\left(\frac{n}{6}\right)^2 t} \sin\frac{n}{6}x. \quad (\textbf{10 points})$$

MATH 220 Quiz 8 Version II Solution **Radford** **12/05/03**

Name (print) ______________________________

Discussion (circle day, time) Tu Th 10 12

(1) *Show* your work for full credit. (2) Give *exact answers* whenever possible; otherwise give answers accurate to two decimal places. (3) *You are expected to abide by the University's rules concerning academic honesty.*

Given that

$$\int x \sin \alpha x \, dx = -\frac{1}{\alpha}(x \cos \alpha x) + \frac{1}{\alpha^2} \sin \alpha x + C$$

and

$$\int x \cos \alpha x \, dx = \frac{1}{\alpha}(x \sin \alpha x) + \frac{1}{\alpha^2} \cos \alpha x + C$$

for $\alpha \neq 0$, find the formal solution to the heat flow problem described by

$$\frac{\partial u}{\partial t} = 7\frac{\partial^2 u}{\partial x^2} \text{ for } 0 < x < 6\pi \text{ and } t > 0, \quad u(0,t) = 0 = u(6\pi, t) \text{ for } t > 0, \quad u(x,0) = x \text{ for } 0 < x < 6\pi.$$

Solution: Generally,

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-\beta\left(\frac{n\pi}{L}\right)^2 t} \sin \frac{n\pi}{L} x.$$

Here $\beta = 7$ and $L = 6\pi$. Therefore $\frac{n\pi}{L} = \frac{n}{6}$ and

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-7\left(\frac{n}{6}\right)^2 t} \sin \frac{n}{6} x. \quad (\textbf{10 points})$$

Using the given we compute

$$b_n = \frac{2}{L}\int_0^{6\pi} x \sin \frac{n}{6} x \, dx = \frac{2}{6\pi}\left[-\frac{1}{\left(\frac{n}{6}\right)} x \cos \frac{n}{6} x + \frac{1}{\left(\frac{n}{6}\right)^2} \sin \frac{n}{6} x\right]_0^{6\pi} = \frac{2}{6\pi}\left(-\frac{6}{n}(6\pi)(\cos n\pi)\right)$$

so

$$b_n = -\frac{12}{n}(-1)^n = \frac{12}{n}(-1)^{n+1}.$$

and therefore

$$u(x,t) = \sum_{n=1}^{\infty} \frac{12}{n}(-1)^{n+1} e^{-7\left(\frac{n}{6}\right)^2 t} \sin \frac{n}{6} x. \quad (\textbf{10 points})$$