Revised Syllabus for the Final and Study Guide.

11/28/04 Radford

The final examination will be on Chapters 1–16, as indicated below. You have a lot of written material to use in your study at this point; see our course home page.

Chapter 0 3–23

- *Basic Concepts*: Well-ordering, induction, divides, divisor, greatest common divisor, Division Algorithm for integers, Euclidean Algorithm, factor, multiple, least common multiple, equivalence relations and classes, functions, one-one, onto, inverse function.
- *Particular Results*:
- *Problems of Note*:
- *Omit*: Examples 5, 6, 8, 10.
- *Comments*: The material of Chapter 0 is considered prerequisite.

Chapter 1 31–37.

- *Basic Concepts*: Cayley table.
- *Particular Results*:
- *Problems of Note*: Construction of Cayley tables for groups.
- *Omit*:
- *Comments*:

Chapter 2 42–52

- *Basic Concepts*: Binary operation, closure, group.
- *Particular Results*:
- *Problems of Note*:

- *Omit*: Examples 14, 20.
- *Comments*: The examples are particularly good. Be sure to understand the table on page 52. Understand how to show closure.

Chapter 3 58–67

- *Basic Concepts*: Order of group, order of element, subgroup, multiplicative and additive notations.
- *Particular Results*: Tests for subgroups, when a finite subset of a group is a subgroup.
- *Problems of Note*: Know how to determine whether or not a subset of a group is a subgroup.
- *Omit*: Example 11.
- *Comments*:

Chapter 4 73–82

- *Basic Concepts*:
- *Particular Results*:
- *Problems of Note*:
- *Omit*: Nothing.
- *Comments*: See the course notes on cyclic groups also. This topic is absolutely fundamental.

Chapter 5 93–103

- *Basic Concepts*: Permutation of a set, the permutation group of a set, symmetric group, decomposition into product of disjoint cycles, n-cycle, odd permutation, even permutation, the alternating group.
- *Particular Results*: Every permutation of a finite set is a product of disjoint cycles, is a product of 2-cycles.
- *Problems of Note*: Find orders of permutations, write permutations as products of disjoint cycles, as products of 2-cycles.

- *Omit*:
- *Comments*:

Chapter 6 118–125

- *Basic Concepts*: Isomorphism of groups.
- *Particular Results*: Cayley's Theorem.
- *Problems of Note*: Know how to show that a function $f : G \longrightarrow G'$ of groups is a group isomorphism, is not a group isomorphism.
- *Omit*: Example 8.
- *Comments*:

Chapter 7 134–138

- *Basic Concepts*: Left coset, right coset, of a subgroup.
- *Particular Results*: Basic properties of left cosets, Lagrange's Theorem and consequences.
- *Problems of Note*: Calculate left, right, cosets.
- *Omit*:
- *Comments*: This material is extremely important.

Chapter 8 150–156

- *Basic Concepts*: External direct product, order of element in direct product.
- *Particular Results*: When a direct product is cyclic.
- *Problems of Note*: Find cyclic subgroups of a direct product of given order.
- *Omit*: Theorem 8.3 to rest of chapter.
- *Comments*:

Chapter 9 172–179

- *Basic Concepts*: Normal subgroup, factor group.

- *Particular Results*:
- *Problems of Note*: Determine whether or not a subgroup is normal.
- *Omit*: Examples 4, 9, 10.
- *Comments*: Cosets may be represented in different ways.

Chapter 10 194–205

- *Basic Concepts*: Group homomorphism, kernel of a group homomorphism,
- *Particular Results*: Kernels are normal subgroups and normal subgroups are normal. The First Isomorphism Theorem for groups.
- *Problems of Note*: Know how to show that a function $f : G \longrightarrow G'$ of groups is a group homomorphism, is not a group homomorphism.
- *Omit*: Examples 11, 12, 15, 16.
- *Comments*: Understand the concept of *well-defined*. The problem of determining all group homomorphisms $f : G \longrightarrow G'$ of finite-cyclic groups is an excellent application of most of the fundamental ideas concerning cyclic groups.

Chapter 12 229–234

- *Basic Concepts*: Ring, ring with unity, units of a ring with unity, the group of units R^* of a ring R with unity, subring, direct sum of rings.
- *Particular Results*:
- *Problems of Note*: Determine whether or not a subset of a ring R is a subring.
- *Omit*:
- *Comments*: Rings are additive abelian groups and subrings are additive subgroups. Thus the theory of rings builds on the theory of abelian groups.

Chapter 13 240–246

- *Basic Concepts*: Zero divisor, integral domain, field, characteristic of a ring with unity.
- *Particular Results*: Fields are integral domains. Finite integral domains are fields. $\mathbf{Z}_n$ is an integral domain (hence a field) if and only if n is prime.
- *Problems of Note*: Determine whether or not a commutative ring with unity is a integral domain, is a field.
- *Omit*:
- *Comments*:

Chapter 14 253–260

- *Basic Concepts*: Ideal, factor ring, prime ideal, maximal ideal.
- *Particular Results*: R a commutative ring with unity, I an ideal of R. Then R/I is an integral domain (resp. field) if and only if I is a prime (resp. maximal) ideal of R.
- *Problems of Note*: Determine whether or not a subset I of a ring R is an ideal of R. Ideals of $\mathbf{Z}$.
- *Omit*: Examples 10, 11.
- *Comments*: Every ideal of a ring is a subring and hence an additive subgroup.

Chapter 15 270–277

- *Basic Concepts*: One-one, onto, ring homomorphism, kernel.
- *Particular Results*: Basic properties of ring homomorphisms, every kernel is an ideal, and every ideal is a kernel, first isomorphism theorem for rings. Every integral domain is a subring of a field (the field of quotients).
- *Problems of Note*: Know how to show that a function $f : R \longrightarrow R'$ of rings is a ring homomorphism, is not a ring homomorphism.
- *Omit*: Examples 8, 9.
- *Comments*: Every ring homomorphism is a homomorphism of additive groups. Thus the theory of ring homomorphisms builds on the theory of group homomorphisms.

Chapter 16 283–290

- *Basic Concepts*: Ring of polynomials $R[x]$ over a commutative ring R, degree of polynomial, monic polynomial, leading coefficient, constant polynomial.
- *Particular Results*: R a division ring implies $R[x]$ is also. The Division Algorithm for $F[x]$. Remainder and Factor Theorems. The structure of ideals of $F[x]$.
- *Problems of Note*: Consequences of an analog of the Euclidean Algorithm for integers. Finding quotients and remainders in $F[x]$ for various fields F.
- *Omit*:
- *Comments*: The ideal structure of $F[x]$ and the subgroup structure of $\mathbf{Z}$ are similar by virtue of the division algorithms.