# Homework #10 (week of 10/25–10/29)

Due Friday, 10/29/04 in class

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1. Let $G$ be a group and $H$ be a subgroup of $G$.

   a) If $[G{:}H] = 2$ show that $H$ is a normal subgroup of $G$.

   b) Suppose that $|H| = 2$. Show that $H$ is a normal subgroup of $G$ if and only if $H \subseteq \mathrm{Z}(G)$.

   c) Determine all normal subgroups of $G = S_3$.

2. Let $G = \mathrm{GL}(n, \boldsymbol{R})$ be the group of all $n \times n$ invertible matrices with real coefficients under matrix multiplication. Suppose that $H$ is a subgroup of $G$. Show that

$$N = \{h \in H \mid \mathrm{Det}(h) = 1\}$$

is a normal subgroup of $H$.

3. Let $G$ be a group and suppose that $\{H_\imath\}_{\imath \in I}$ is an indexed family of subgroups of $G$. Then $H = \cap_{\imath \in I} H_\imath$ is a subgroup of $G$.

Suppose that $H_\imath$ is a normal subgroup of $G$ for all $\imath \in I$. Show that $H$ is a normal subgroup of $G$.

4. Let $G = S_n$ and $\sigma = (a_1 \;\; a_2 \;\; \cdots \;\; a_m) \in G$ be an $m$-cycle. Then for $\tau \in G$ the element $\tau\sigma\tau^{-1}$ is an $m$-cycle given by

$$\tau\sigma\tau^{-1} = (\tau(a_1) \;\; \tau(a_2) \;\; \cdots \;\; \tau(a_m)). \tag{1}$$

Use (1) to find all normal subgroups of $G = S_4$.