Homework # 4 (week of 09/13–09/17)

Due Friday, 09/17/04 in class

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1. Let G be a group.

   a) Suppose that H, K, are subgroups of G. Show that $H \cap K$ is a subgroup of G.

   b) Suppose that $H_1, \ldots, H_n$ are subgroups of G. Show, by induction, that $H_1 \cap \cdots \cap H_n$ is a subgroup of G.

2. For the groups $G = \boldsymbol{Z}_{12}$ and $G = U(12)$ do the following:

   a) For each $a \in G$ list the elements of $<a>$;

   b) For each $a \in G$ find the order of a;

   c) Determine whether or not G is cyclic.

3. Let G be a group.

   a) Suppose that $a^2 = e$ for all $a \in G$. Show that G is abelian. [Hint: For $a, b \in G$ consider $(ab)^2$.]

   b) Let $a \in G$. Show that $a = a^{-1}$ if and only if $a^2 = e$.

   c) Suppose that G is finite and $x^2 = e$ has a unique solution in G (which must be $x = e$.) Show that $|G|$ is odd. [Hint: Show that $x \sim y$ if and only if $x = y$ or $x = y^{-1}$ is an equivalwnce relation on G. Note that the equivalence class $[x] = \{x, x^{-1}\}$ for all $x \in G$ has one or two elements.]

4. Let $\boldsymbol{Q}$ be the group of all rational numbers under adddition and let $\boldsymbol{Q}^*$ be the group of all non-zero rational numbers under multiplication.

   a) Describe the elements of the subgroup $<7>$ in $\boldsymbol{Q}$.

   b) Describe the elements of the subgroup $<7>$ in $\boldsymbol{Q}^*$.