Homework # 6 (week of 09/27–10/01)

Due Friday, 10/01/04 in class

---

1. Let $G = <a>$ be a cyclic group of order 308 and $H = <a^{140}>$.

   a) Find a positive divisor d of 308 such that $H = <a^{140}> = <a^d>$ and determine $|H|$.

   b) List all of the generators of H as a^ℓ, where $1 \leq \ell \leq 308$.

   c) Describe the distinct subgroups of H as $<a^\ell>$, where $0 \leq \ell \leq 308$.

   d) Construct a lattice diagram for G, and one for H.

2. Let $G = <a>$ be a cyclic group of order pqr, where p, q, r are distinct positive prime integers. Write down a lattice diagram for G, and indicate the order of each subgroup of G.

3. For each of the two permutations σ of S_9 listed below do the following

   a) Write σ as a product of disjoint cycles;

   b) Write σ as a product of transpositions;

   c) Determine the order of σ.

$$\sigma = (1\ 3)(5\ 4\ 6)(4\ 2\ 6)(5\ 2\ 3)(7\ 9\ 2), \qquad \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 1 & 9 & 8 & 2 & 5 & 7 & 3 \end{pmatrix}.$$

4. Let $G = S_n$ where $n \geq 1$, let H be a subgroup of G. Then the intersection of subgroups $K = A_n \cap H$ is a subgroup of H. Suppose that $\tau \in H$ is an odd permutation and let $\tau K = \{\tau\sigma \mid \sigma \in K\}$.

   a) Show that the function $F : K \longrightarrow \tau K$ defined by $F(\sigma) = \tau\sigma$ for all $\sigma \in K$ is a one-one onto function.

b) Show that $H = K \cup \tau K$ and $K \cap \tau K = \emptyset$.

c) Show that $|K| = |\tau K|$ and $|H| = 2|K|$. (Thus $|H|$ is even and the number of even permutations of H is $|H|/2$.)

d) Show that a subgroup of G of odd order consists of even permutations.