

# Homework #9 (week of 10/18–10/22)

Due Friday, 10/22/04 in class

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1. For each group below find a direct sum  $\mathbf{Z}_{n_1} \oplus \cdots \oplus \mathbf{Z}_{n_r}$  isomorphic to it, where  $1 < n_1$  and  $n_1 | n_2, \dots, n_{r-1} | n_r$ :

a)  $\mathbf{Z}_2 \oplus \mathbf{Z}_4 \oplus \mathbf{Z}_6$ ;

b)  $\mathbf{Z}_4 \oplus \mathbf{Z}_6 \oplus \mathbf{Z}_8$ ;

c)  $\mathbf{Z}_{10} \oplus \mathbf{Z}_{12} \oplus \mathbf{Z}_{14}$ .

2. For each of the groups of Problem 1 find a direct sum  $\mathbf{Z}_{p_1^{n_1}} \oplus \cdots \oplus \mathbf{Z}_{p_r^{n_r}}$  isomorphic to it, where  $1 \leq n_1, \dots, n_r$  and  $p_1, \dots, p_r$  are prime integers.

3. Let  $G = H \times K$  be the external direct product of two groups and suppose that  $L$  is a subgroup of  $G$ . Show that

$$H' = \{h \in H \mid (h, k) \in L \text{ for some } k \in K\}$$

is a subgroup of  $H$ .

4. Let  $G = \mathbf{Z}_p \oplus \mathbf{Z}_{p^2}$  where  $p$  is a prime integer. Determine all of the non-cyclic subgroups of  $G$ . [Hint: Use the preceding exercise.]