MATH 330, MTHT 435 Hour Exam I **Radford** **10/08/04**

Name (print) ______________________________

(1) There are *four questions* on this exam. (2) You may *keep* this exam copy. (3) You do *not* need to show how associativity is used in proofs. (4) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (20 pts.) Let G be a group and suppose that $\{H_\imath\}_{\imath \in I}$ is a family of subgroups of G. Show that
$$K = \cap_{\imath \in I} H_\imath = \{g \in G \mid g \in H_\imath \text{ for all } \imath \in I\}$$
is a subgroup of G.

2. (30 pts.) Let $G = \lt a \gt$ be a cyclic group of order 45.
   a) How many subgroups does G have?
   b) For each subgroup of G list its size and *one* generator in the form a^ℓ, where $0 \leq \ell < 45$.
   c) List *all* of the generators of the subgroup of G of order 15 in the form a^ℓ, where $0 \leq \ell < 45$.
   d) Find a divisor d of 45 such that $\lt a^{250} \gt = \lt a^d \gt$ and list the distinct elements of $\lt a^{250} \gt$ in the form a^ℓ, where $0 \leq \ell < 45$.
   e) Draw a lattice diagram for G.

3. (25 pts.) Let $G = \mathrm{GL}(2, \boldsymbol{R})$ be the group of 2×2 matrices with real coefficients under matrix multiplication. Let $H = \{\begin{pmatrix} 2^m & b \\ 0 & 3^n \end{pmatrix} \mid m, n \in \boldsymbol{Z}, b \in \boldsymbol{R}\}$. Show that H is a subgroup of G.

4. (25 pts.) Consider the permutation $f = (1\,3\,5\,4\,6)(2\,4\,6\,9\,7)(7\,8\,9)$ of S_9.
   a) Write f as a product of *disjoint* cycles.
   b) Write f as a product of transpositions (2-cycles).
   c) Is f even? You must justify your answer.
   d) Write f^2 as a product of disjoint cycles.