MATH 330, MTHT 435 # Hour Exam II **Radford 11/19/04**

Name (print) _____

(1) There are *five questions* on this exam. (2) You may *keep* this exam copy. (3) You do *not* need to show how associativity is used in proofs. (4) *You are expected to abide by the University's rules concerning academic honesty.*

1. (20 points) Let $G = \mathrm{A}_4$ and $H = <(1\,2\,3)>$.

   a) How many left cosets does H have in G?

   b) List the elements of $(1\,2)(3\,4)H$ as products of disjoint cycles.

   c) Determine whether or not H a normal subgroup of G.

2. (25 points) Let $G = \mathbf{Z}_4 \oplus \mathbf{Z}_6$

   a) According to Lagrange's Theorem, what are the possible orders of subgroups of G?

   b) Find two different cyclic subgroups of G of order 6.

   c) Find all generators of cyclic subgroups of G of order 12. Your answer must be justified.

   d) Find a non-cyclic subgroup of G of order 12. Your answer must be justified. [Hint: You may assume that the product of subgroups of groups G' and G'' respectively is a subgroup of the product $G' \oplus G''$.]

3. (20 points) Let R be a commutative ring with unity.

   a) For $a \in R$ show that $Ra = \{ra \mid r \in R\}$ is an ideal of R.

   b) Suppose that R is a field. Show that $Ra = R$ for all non-zero $a \in R$.

4. (15 points) Let $\mathbf{C}$ be the field of complex numbers and $R = \mathrm{M}(2, \mathbf{R})$ be the ring of 2×2 matrices with real coefficients under matrix addition and multiplication. Show that $\phi : \mathbf{C} \longrightarrow R$ defined by $\phi(r + si) = \begin{pmatrix} r & -s \\ s & r \end{pmatrix}$ for all $r + si \in \mathbf{C}$ is a one-one ring homomorphism. (Thus $\mathbf{C}$ can be realized as a subring of 2×2 matrices with real coefficients.)

5. (20 points) Let $R = \mathrm{M}(2, \mathbf{R})$ be the ring of 2×2 matrices with real coefficients under matrix addition and multiplication and let $S = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, d \in \mathbf{Z}, b, c \in 2\mathbf{Z}\}$.

   a) Show that S is a subring of R.

   b) Determine whether or not R is an integral domain.