Solution to Homework # 1

09/03/03 Radford

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1. (page 23, number 2) (**10 points total**).

$$\begin{aligned}\gcd(2^4{\cdot}3^2{\cdot}5{\cdot}7^2,\ 2{\cdot}3^3{\cdot}7{\cdot}11) &= 2^{\min(4,1)}{\cdot}3^{\min(2,3)}{\cdot}5^{\min(1,0)}{\cdot}7^{\min(2,1)}{\cdot}11^{\min(0,1)}\\ &= 2^1{\cdot}3^2{\cdot}5^0{\cdot}7^1{\cdot}11^0\\ &= \boxed{2{\cdot}3^2{\cdot}7}\ \ (\mathbf{5\ \ points})\end{aligned}$$

and

$$\begin{aligned}\mathrm{lcm}(2^3{\cdot}3^2{\cdot}5,\ 2{\cdot}3^3{\cdot}7{\cdot}11) &= 2^{\max(3,1)}{\cdot}3^{\max(2,3)}{\cdot}5^{\max(1,0)}{\cdot}7^{\max(0,1)}{\cdot}11^{\mathrm{maxn}(0,1)}\\ &= 2^3{\cdot}3^3{\cdot}5^1{\cdot}7^2{\cdot}11^1\\ &= \boxed{2^3{\cdot}3^3{\cdot}5{\cdot}7{\cdot}11}\ \ (\mathbf{5\ \ points}).\end{aligned}$$

2. (page 23, number 6) (**10 points total**). By guessing we see that

$$248 = (63)(3) + (2)(29) + 1$$

and

$$601 = (63)(9) + (2)(17).$$

Thus the person with digits 248 is $\boxed{\text{a female born on March 29}}$ (**5 points**) and the person with digits 601 is $\boxed{\text{a male born on September 17}}$ (**5 points**).

3. (page 24, number 16) (**10 points total**). We first find the greatest common divisor of 126 and 34 by the Euclidean algorithm (**3 points for calculation**).

$$\begin{aligned}\mathbf{126} &= 3{\cdot}\mathbf{34} + 24\\ \mathbf{34} &= 1{\cdot}\mathbf{24} + 10\\ \mathbf{24} &= 2{\cdot}\mathbf{10} + 4\\ \mathbf{10} &= 2{\cdot}\mathbf{4} + 2\\ \mathbf{4} &= 2{\cdot}\mathbf{2} + 0\end{aligned}$$

Therefore $\boxed{\gcd(126, 34) = 2}$ (**3 points**). We start with the second to the last equation and "back substitute":

$$\begin{aligned} \mathbf{2} &= \mathbf{10} - 2 \cdot \mathbf{4} \\ &= \mathbf{10} - 2 \cdot (\mathbf{24} - 2 \cdot \mathbf{10}) \\ &= 5 \cdot \mathbf{10} - 2 \cdot \mathbf{24} \\ &= 5 \cdot (\mathbf{34} - 1 \cdot \mathbf{24}) - 2 \cdot \mathbf{24} \\ &= 5 \cdot \mathbf{34} - 7 \cdot \mathbf{24} \\ &= 5 \cdot \mathbf{34} - 7 \cdot (\mathbf{126} - 3 \cdot \mathbf{34}) \\ &= (-7) \cdot \mathbf{126} + 26 \cdot \mathbf{34}. \end{aligned}$$

One can check that $\boxed{2 = (-7) \cdot 126 + 26 \cdot 34}$. (**4 points**).

4. (page 24, number 18) (**10 points total**). Let $p_1, \ldots, p_n$ be prime integers and $q = p_1 \cdots p_n + 1$. We will show that $p_\imath$ does not divide q for *all* $1 \leq \imath \leq n$ by proof by contradiction.

$\boxed{\text{Suppose that our assertion is false. Then } p_\imath \text{ divides } q \text{ for } \textit{some } 1 \leq \imath \leq n}$ (**2 points**). Fix such an $\imath$. Since $p_\imath$ divides q by definition $q = ap_\imath$ for some integer a. Now $p_\imath$ divides the product $p_1 \cdots p_n$ since $p_1 \cdots p_n = bp_\imath$, where b is the product with the factor $p_\imath$ omitted. Thus

$$\boxed{ap_\imath = q = p_1 \cdots p_n + 1 = bp_\imath - 1} \quad (\mathbf{3 \ points})$$

which implies that

$$\boxed{1 = ap_\imath - bp_\imath = (a - b)p_\imath} \quad (\mathbf{3 \ points}).$$

$\boxed{\text{Since } a - b \text{ and } p_\imath \text{ are integers, and } p_\imath \neq \pm 1 \text{, this is impossible.}}$ (**2 points**). Therefore our assumption that the assertion is false is itself false. Thus our assertion is ture.