Homework # 3 (week of 09/06–09/10)

Due Friday, 09/10/04 in class

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1. Let $G = \mathrm{GL}(2, \boldsymbol{R})$ be the group of 2×2 invertible matrices with real coefficients under matrix multiplication.

   a) Show that G is not abelian by finding specific $g, g' \in G$ such that $gg' \neq g'g$.

   b) Find all $g \in G$ such that $gx = xg$ for all $x \in G$.

   c) Determine whether or not $H = \{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in \boldsymbol{R}, ac \neq 0 \}$ is a subgroup of G.

2. Let $G = U(9)$.

   a) Write out the Cayley table for G.

   b) Find a $g \in G$ such that $G = \langle g \rangle$.

3. Prove that $G = \{ \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \mid a, b, c \in \boldsymbol{R} \}$ is a group under matrix multiplication. (You may assume that matrix multiplication is associative.)