Math 160 Sp'10 Lowman Lecture: Monday Week 11

Note Title 4/5/2010

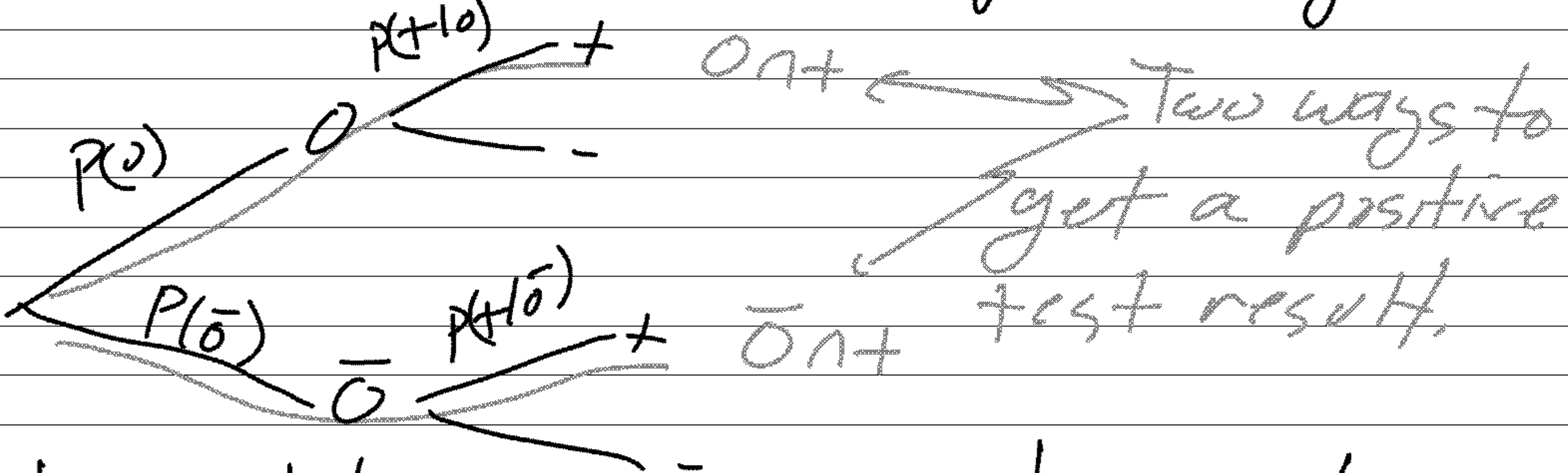
Bayes' Theorem:

The oil drilling problem from the last lecture is an example of a Bayes Theorem type of problem. The given conditional probabilities were of form: $P(+|O)$ and $P(-|O)$. However, the probabilities to be found were of form $P(O|+)$

ie $P(O|+) = \frac{P(O\cap +)}{P(+)} = \frac{P(O\cap +)}{P(O\cap +)+P(\bar{O}\cap +)}$

$$= \frac{P(O)\cdot P(+|O)}{P(O)\cdot P(+|O) + P(\bar{O})\cdot P(+|\bar{O})}$$

This is a special case of Bayes' formula. The probabilities were found from a tree and Bayes' formula was not explicitely used.



When there many branches in

the 1st stage of the tree it is often convenient to use a Bayes' Formula.

Here is a simple example that demonstrates how to derive Bayes' Formula.

Assume a factory has three

machines that make parts: m_1, m_2, m_3
Each machine makes defective (D) parts and good parts ($\bar{D}$). From past records these probabilities are known:

$P(m_1)$ probability (fraction) of parts made by machine 1.

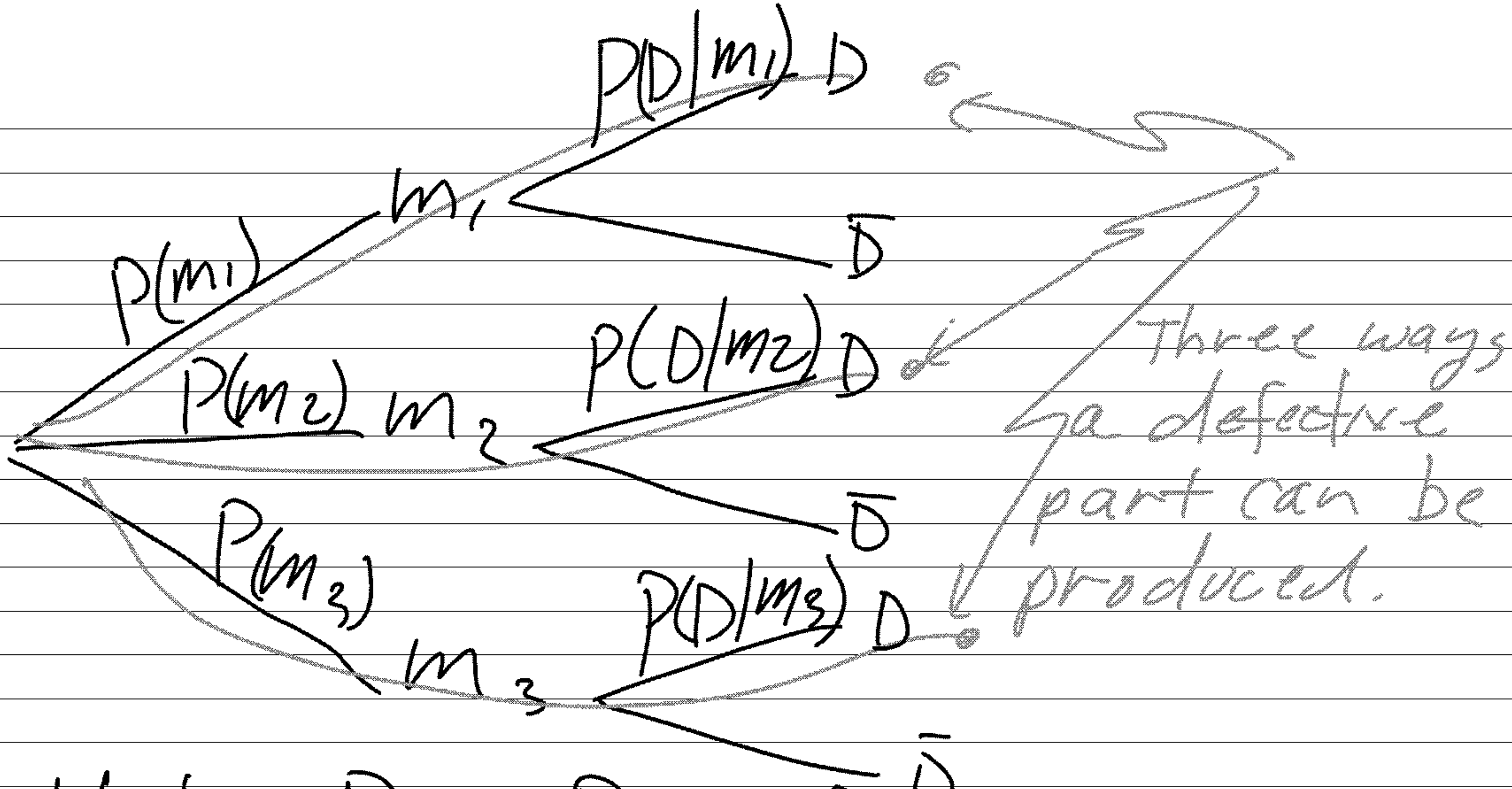
$P(m_2)$ probability a part is made by m_2

$P(m_3)$ fraction of parts made by m_3.

It is also known what the probabilities are that each machine will produce a defective part

$P(D|m_1)$, $P(D|m_2)$, $P(D|m_3)$

A tree can be used to organize the information.



Note: $P(m_1) + P(m_2) + P(m_3) = 1$

Now, at the end of the day a part is chosen at random and it is defective. What is the probability that it was made by machine 1?

From the tree and the definition of conditional probability

$$P(D|m_1) = \frac{P(m_1 \cap D)}{P(D)} = \frac{P(m_1 \cap D)}{P(m_1 \cap D) + P(m_2 \cap D) + P(m_3 \cap D)}$$

$$= \frac{P(m_1) \cdot P(D|m_1)}{P(m_1) \cdot P(D|m_1) + P(m_2) \cdot P(D|m_2) + P(m_3) \cdot P(D|m_3)}$$

$$= \frac{P(m_1) \cdot P(D|m_1)}{\sum_{i=1}^{3} P(m_i) \cdot P D|m_i}$$

You can ask the same question for machines $i=1$ or $i=2$ or $i=3$. giving a more general formula

Bayes' Formula

$$P(m_i \mid D) = \frac{P(m_i) \cdot P(D \mid m_i)}{\sum_{i=1}^{n} P(m_i) \cdot P(D \mid m_i)}$$

If you remember and understand this problem then you can easily reproduce Bayes' Formula when you need it. In addition, have a good sense for how to use it.

Here are two practice problems for probability. They are not Bayes' problems.

Example: Awards will be given to seven people whose names start with letters J, K, L, M, N, O and P. In how many ways can the awards be presented if the second award is presented to either K, J or N?

In counting problems where there restrictions, it is usually best to follow a process that takes care of the restrictions first. This usually simplifies the problem.

Process: Assign people to the awards one thru seven

___ ___ ___ ___ ___ ___ ___
1st 2nd 3rd 4th 5th 6th 7th.

1st Assign a person to the 2nd award. there are 3 ways to do this

2nd There are now 6 people to be assigned to the 6 remains

awards. This can be done in
$6!$ wayes.

$$\boxed{N = 3 \cdot 6!}$$

Example: Two year old Andy helped his mother unpack groceries. In the process he removed the labels from all of the cans. Five cans contained peaches, six cans contained vegetables, five contained soup, and three contained dog food. His mother plans to open three cans for dinner. What is the probability that she will open exactly one can of vegetables?

A good way to start is to draw a picture.

5	Peaches
6	Veggies
5	Soup
3	Dog Food

$N = 19$

Pick 3 at random. Order does not matter.

In this case the problem can be simplified because the question only asks about getting veggies OR Not. Redraw the box.

a) $\boxed{\begin{array}{ll} 6 & V \\ 13 & \overline{V} \end{array}} \to$ Pick 3 at Random.

$N = 19$

P(only one can of Veggies)

$= P(1V \text{ and } 2\overline{V}) = \dfrac{n(1V \text{ and } 2\overline{V})}{n(\text{any 3 cans})}$

$= \dfrac{n(1V) \cdot n(2\overline{V})}{n(\text{any 3 from 19})}$

$$= \frac{\binom{6}{1} \cdot \binom{13}{2}}{\binom{19}{3}} = \underline{\hspace{4cm}}$$

(finish the calculation).