- Matrix multiplication $\mathbf{A} \cdot \mathbf{B}$ is defined only if the number of columns of matrix $\mathbf{A}$ is the same as the number of rows of matrix $\mathbf{B}$.
- $\mathbf{A_{mxp}} \cdot \mathbf{B_{pxn}} = \mathbf{C_{mxn}}$, $\mathbf{p}$ is the number of columns of $\mathbf{A}$ and $\mathbf{p}$ is the number of rows of $\mathbf{B}$.
- The resulting matrix $\mathbf{C}$ has the same number of rows as $\mathbf{A}$ and the same number of columns as $\mathbf{B}$
- Matrix multiplication is defined in terms of multiplying a row matrix by a column matrix. Both matrices must have the same number of elements.

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \end{bmatrix} \text{ and } , \mathbf{B} = \begin{bmatrix} \mathbf{3} \\ \mathbf{4} \end{bmatrix} \tag{1}$$

$$\mathbf{A} \cdot \mathbf{B} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{3} \\ \mathbf{4} \end{bmatrix} = \begin{bmatrix} \mathbf{1} \cdot \mathbf{3} + \mathbf{2} \cdot \mathbf{4} \end{bmatrix} = \begin{bmatrix} \mathbf{11} \end{bmatrix} \tag{2}$$

$$\mathbf{C} = \mathbf{A} \cdot \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 20 & 13 \end{bmatrix} \tag{3}$$

$\mathbf{c_{ij}}$ is the element in row $\mathbf{i}$ and column $\mathbf{j}$ of matrix $\mathbf{C} = \mathbf{AB}$
$\mathbf{c_{ij}}$ is found by multiplying row $\mathbf{i}$ of matrix $\mathbf{A}$ by column $\mathbf{j}$ of matrix $\mathbf{B}$

$$\mathbf{c_{ij}} = \begin{bmatrix} \text{ith row of A} \end{bmatrix} \cdot \begin{bmatrix} \text{jth} \\ \text{column} \\ \text{of B} \end{bmatrix}$$

$$c_{11} = \begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 8, \quad c_{12} = \begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 5 \tag{4}$$

$$c_{21} = \begin{bmatrix} 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 20, \quad c_{22} = \begin{bmatrix} 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 13 \tag{5}$$

Write the system of equations in matrix form.

$$x + 2y = 5 \quad (6)$$

$$3x + 4y = 19 \quad (7)$$

$$\begin{bmatrix} x + 2y \\ 3x + 4y \end{bmatrix}_{2x1} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}_{2x1} \quad (8)$$

$$(9)$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2x2} \cdot \begin{bmatrix} x \\ y \end{bmatrix}_{2x1} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}_{2x1} \quad (10)$$

$$(11)$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix} \quad (12)$$

$$(13)$$

$$A \cdot X = B \quad (14)$$

Three cases:

- $\mathbf{AB}$ is defined but $\mathbf{BA}$ is not defined.
- $\mathbf{AB}$ and $\mathbf{BA}$ are both defined but not the same
- $\mathbf{AB}$ does equal $\mathbf{BA}$
- Summary: In general $\mathbf{AB} \neq \mathbf{BA}$ but in some cases it is true that $\mathbf{AB} = \mathbf{BA}$