Linear Programming

example: problem setup

- A truck traveling from New York to Baltimore is to be loaded with two types of cargo. Each crate of cargo A is 5 cubic feet in volume, weighs 100 pounds, and earns $12 for the driver. Each crate of cargo B is 3 cubic feet in volume, weighs 25 pounds, and earns $7 for the driver. The truck can carry no more than 300 cubic feet of crates and no more than 1,000 pounds (half-ton pickup truck). Also, the number of crates of cargo B must be less than or equal to twice the number of crates of cargo A.

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- Setup the linear programming problem to maximize the drivers earnings.
  - Find all linear constraints and the objective function.
  - Use the corner-point method to solve.

Linear Programming
organize given information

A truck traveling from New York to Baltimore is to be loaded with two types of cargo.

- crate A
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- another restriction
  - number of creates of cargo B must be less than or equal to twice the number of crates of cargo A

Linear Programming
setup problem

- let x be number of crates of A

Linear Programming

setup problem

- let x be number of crates of A
- let y be the number of crates of B

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Linear Programming
setup problem

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- let y be the the number of crates of B
- **Problem:** maximize drivers earnings subject to constraints
  - maximize $\mathbf{z = \$12x + \$7y}$
  - subject to constraints.

Constraint: The truck can carry no more than 300 cubic feet of crates.

- $\mathbf{(}$ cubic feet of A$\mathbf{)} + \mathbf{(}$cubic feet of B$\mathbf{)} \leq \mathbf{300ft^3}$

Linear Programming

setup constraints-1

Constraint: The truck can carry no more than 300 cubic feet of crates.

- **(** cubic feet of A**)** + **(**cubic feet of B**)** $\leq$ **300ft^3**
- **(**$\frac{\mathbf{5ft^3}}{\mathbf{1crate}}$ $\cdot$ num crates A**)** + **(**$\frac{\mathbf{3ft^3}}{\mathbf{1crate}}$ $\cdot$ num crates B**)** $\leq$ **300ft^3**

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- $\boxed{5 \cdot x + 3 \cdot y \leq 300}$

Constraint: The truck can carry no more than 1,000 pounds.

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Linear Programming

setup constraints-2

Constraint: The truck can carry no more than 1,000 pounds.

- (weight of A) + (weight of B) $\leq \mathbf{1,000lbs}$
- $\left(\frac{\mathbf{100lbs}}{\mathbf{1crate}} \cdot \text{num crates A}\right) + \left(\frac{\mathbf{25lbs}}{\mathbf{1crate}} \cdot \text{num crates B}\right) \leq \mathbf{1,000lbs}$
- $\boxed{\mathbf{100 \cdot x + 25 \cdot y \leq 1,000}}$

Linear Programming
setup constraints-3

Constraint: number of crates of cargo B must be less than or equal to twice the number of crates of cargo A.

- $\mathbf{y \leq 2 \cdot x}$

Constraint: number of crates of cargo B must be less than or equal to twice the number of crates of cargo A.

- $\mathbf{y \leq 2 \cdot x}$
- $\boxed{\mathbf{-2x + y \leq 0}}$

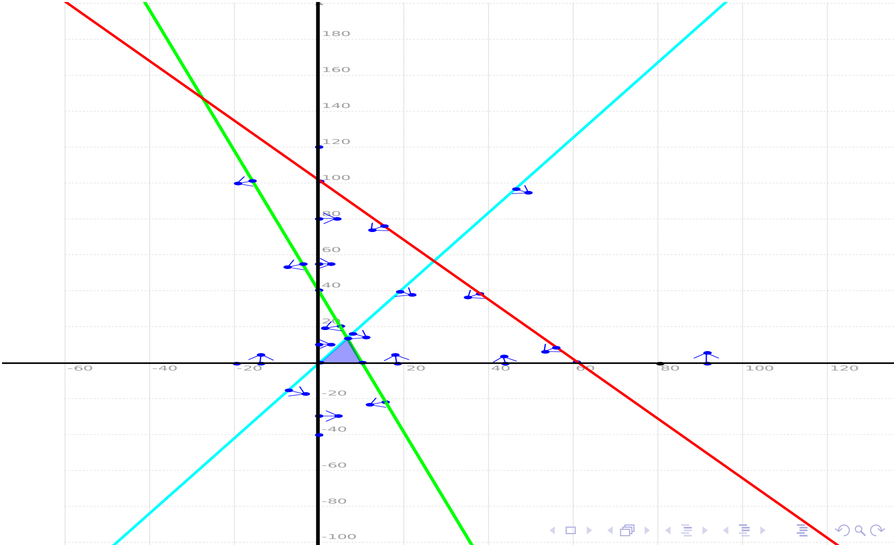
Linear Programming
problem summary

Solve the following linear programming problem by the corner-point method.

$$\begin{aligned}
\text{Maximize: } z &= \$12x + \$7y && (1)\\
\text{subject to: } 5 \cdot x + 3 \cdot y &\leq 300 && (2)\\
100 \cdot x + 25 \cdot y &\leq 1,000 && (3)\\
-2x + y &\leq 0 && (4)\\
x &\geq 0 && (5)\\
y &\geq 0 && (6)
\end{aligned}$$

Linear Programming

graph of constraints



Solve the following linear programming problem by the corner-point method.

$$\text{Maximize: } \mathbf{z = 20x + 15y} \tag{7}$$

$$\text{subject to: } \mathbf{3x + 4y \leq 60} \tag{8}$$

$$\mathbf{4x + 3y \leq 60} \tag{9}$$

$$\mathbf{x \leq 10} \tag{10}$$

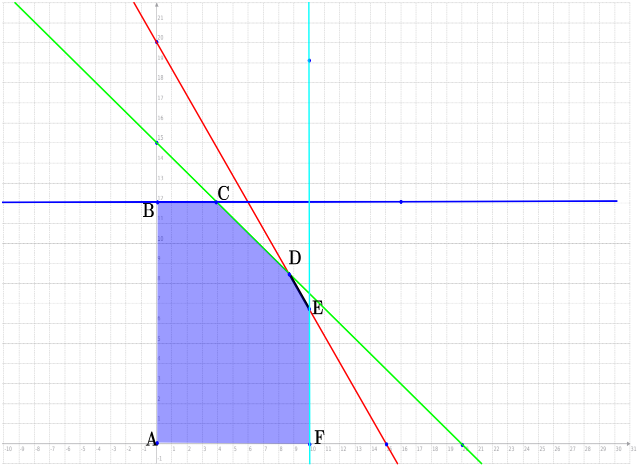
$$\mathbf{y \leq 12} \tag{11}$$

$$\mathbf{x \geq 0} \tag{12}$$

$$\mathbf{y \geq 0} \tag{13}$$

Linear Programming

graph of constraints



Corner Point	(x, y)	$z = 20x + 15y$
A	$(0, 0)$	$20(0) + 15(0) = 0$
B	$(0, 12)$	$20(0) + 15(12) = 180$
C	$(4, 12)$	$20(4) + 15(12) = 260$
D	$(\frac{60}{7}, \frac{60}{7})$	$20(\frac{60}{7}) + 15(\frac{60}{7}) = 300*$
E	$(10, \frac{20}{3})$	$20(10) + 15(\frac{20}{3}) = 300*$
F	$(10, 0)$	$20(10) + 15(0) = 200$

Note the tie for the highest value of z between points D and E. All points on the line segment between D and E will give the same value for for z as at D and E. There are an infinite number of optimal solutions on $\overline{\mathbf{AD}}$